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INVESTIGATION OF STABILITY AND CONTROL CHARACTERISTICS OF AC130 LINEAR MODELS

Robert G. Lorenz

Air Force Institute of Technology
Wright-Patterson Air Force Base, Ohio

March 1972

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13. ABSTRACT

Mathematical models of the AC130A and AC130E aircraft are proposed. The models are developed from linearized equations and are referred to trim conditions of level turning flight.

The proposed AC130E model is compared to an existing model to ascertain whether any significant differences exist between the two. A qualitative comparison is conducted by investigating each model's response to control deflections.

The proposed AC130A model is used to predict general trends and probable values for stability derivatives and selected mode parameters over an extensive flight envelope. ()

The proposed AC130E model exhibited increased phugoid damping, and its dutch roll oscillations and divergent modes were generally weaker than those of the existing model. Short period characteristics were identical.

The data compiled to estimate trends and values of stability parameters for the AC130A aircraft produced no unusual results.

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THESIS

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Capt USAF

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INVESTIGATION OF STABILITY AND CONTROL CHARACTERISTICS OF AC130 LINEAR MODELS

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirement for the Degree of
Master of Science

by

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Preface

This study was undertaken for the Control Systems Development Branch, Air Force Flight Dynamics Laboratory (Flight Control Division). The first objective of the effort was to ascertain whether the existing linear mathematical model of the AC130 was accurate. I thought it best to begin with the basic equations of motion and develop a parallel model, the characteristics of which could be compared to those of the existing model.

Using the model developed in this study, the second purpose was to compile data on selected stability and control parameters for the AC130A aircraft to predict general trends and values for these parameters throughout an extensive flight envelope.

I wish to express my gratitude to the following personnel: to Lt. Col. James Thompson, for sponsoring this study; to Capt. Kenneth Bassett, for supporting the project and guiding my efforts to produce useful results; to Capt. Len Kruczynski, for supplying the basic computer routines used in the computations; and to my wife, for her unending patience with the author.

Robert G. Lorenz

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Notation

Symbol	Description	Units
A	Altitude	ft
a	Speed of sound	ft/sec
A.C.	Aerodynamic center, % MAC	
AR	Aspect ratio	
b	Span	ft
C.G.	Center of gravity, % MAC	
c	Section chord	ft
$\bar{c}$	Mean aerodynamic chord (MAC)	ft
D	Drag	lbf
d	Diameter of propeller	ft
F	Flap position in %	
g	Acceleration due to gravity	ft/sec^2
h_x, h_y, h_z	Scaler components of angular momentum of spinning rotors	slug ft^2/sec
I	Total moment of inertia of propeller	slug ft^2
I_x	Moment of inertia	slug ft^2
I_y	Moment of inertia	slug ft^2
I_z	Moment of inertia	slug ft^2
I_{xz}	Product of inertia	slug ft^2
I_{xy}	Product of inertia	slug ft^2
I_{yz}	Product of inertia	slug ft^2
i	Angle of incidence	deg
K	Induced drag coefficient, $1/\pi eAR$	
L	Lift	lbf

$\mathcal{L}$	Rolling moment about mass center	lbf ft
l_t	Longitudinal distance between center of gravity and of aerodynamic center horizontal tail (assumed to be at $.25\bar{c}$)	ft
l_f	Longitudinal distance between center of gravity and of aerodynamic center vertical tail (assumed to be at $.25\bar{c}$)	ft
M	Mach number	ft
MAC	Mean aerodynamic chord, $\bar{c}$	ft
$\mathcal{M}$	Pitching moment about mass center	lbf ft
m	Mass	slug
$\mathcal{N}$	Yawing moment about mass center	lbf ft
P,Q,R	Scaler components of angular velocity	rad/sec
p,q,r	Perturbations of P,Q,R	rad/sec
$\hat{p},\hat{q},\hat{r}$	Nondimensionalized perturbations, $pb/2U_o$, $q\bar{c}/2U_o$, $rb/2U_o$	
R	Radius of turn	ft
S	Total wing area	ft^2
S_t	Total area of horizontal tail	ft^2
TAS	True air speed	ft/sec
T	Thrust	lbf
T_c	Specific thrust, $T(eng)/\rho U^2 d^2$, where d is propeller diameter	
U,V,W	Scaler components of V_c	ft/sec
u,v,w	Perturbation of U,V.W	ft/sec
$\hat{u},\hat{v},\hat{w}$	Nondimensionalized perturbations, u/U_o,v/U_o,w/U_o	
V_c	Velocity of mass center	ft/sec
X,Y,Z	Scaler components of aerodynamic force acting on mass center	lbf

x, y, z	Displacements along axes	ft
α	Angle of attack (see Fig. 1)	rad
α_w	Angle of attack of wing, $\alpha + i_w$	rad
β	Sideslip angle (see Fig. 1)	rad
$\delta_a, \delta_e, \delta_r$	Perturbation angles of ailerons, elevator, and rudder	rad
ρ	Air density	slug/ft^3
Θ, Φ, Ψ	Euler angles	rad
θ, ϕ, ψ	Perturbations of Θ, Φ, Ψ	rad
Ω	Propeller RPM	rad/sec
ξ	Damping ratio	

Subscripts

a	Aileron
e	Elevator
f	Vertical tail
i	Inertial
o	Denotes evaluation at trim condition (except for C_{L_o}, C_{d_o})
r	Rudder
t	Horizontal tail
T	Component due to thrust
w	Wing

	Description
	Nondimensional Coefficients
C_D	$(D/1/2\rho SV_c^2)_o$
C_{D_o}	$(C_D$ at $C_L=0)$
C_{D_α}	$(\partial C_D/\partial\alpha)_o$
C_L	$(L/1/2\rho SV_c^2)_o$
C_{L_o}	$(C_L$ at $\alpha=0)$
C_{L_α}	$(\partial C_L/\partial\alpha)_o$
C_l	$(\mathcal{L}/1/2\rho SbV_c^2)_o$
C_{l_p}	$(\partial C_l/\partial\hat{p})_o$
C_{l_r}	$(\partial C_l/\partial\hat{r})_o$
C_{l_β}	$(\partial C_l/\partial\beta)_o$
$C_{l_{\delta a}}$	$(\partial C_l/\partial\delta a)_o$
$C_{l_{\delta r}}$	$(\partial C_l/\partial\delta r)_o$
C_m	$(\mathcal{M}/1/2\rho ScV_c^2)_o$
C_{m_q}	$(\partial C_m/\partial\hat{q})_o$
C_{m_u}	$(\partial C_m/\partial\hat{u})_o$
C_{m_α}	$(\partial C_m/\partial\alpha)_o$
$C_{m_{\dot\alpha}}$	$[\partial C_m/\partial(\dot\alpha\bar{c}/2U_o)]_o$
$C_{m_{\delta e}}$	$(\partial C_m/\partial\delta e)_o$
C_n	$(\mathcal{N}/1/2\rho SbV_c^2)_o$
C_{n_p}	$(\partial C_n/\partial\hat{p})_o$
C_{n_r}	$(\partial C_n/\partial\hat{r})_o$
C_{n_β}	$(\partial C_n/\partial\beta)_o$
$C_{n_{\delta a}}$	$(\partial C_n/\partial\delta a)_o$
$C_{n_{\delta r}}$	$(\partial C_n/\partial\delta r)_o$
C_T	$(T/1/2\rho SV_c^2)_o$
C_x	$(X/1/2\rho SV_c^2)_o$
C_{x_q}	$(\partial C_x/\partial\hat{q})_o$
C_{x_u}	$(\partial C_x/\partial\hat{u})_o$
C_{x_α}	$(\partial C_x/\partial\alpha)_o$
$C_{x_{\dot\alpha}}$	$[\partial C_x/\partial(\dot\alpha\bar{c}/2U_o)]_o$

$C_{x_{\delta e}}$	$(\partial C_x/\partial \delta e)_o$
C_y	$(Y/1/2\rho S V_c^2)_o$
C_{y_p}	$(\partial C_y/\partial \hat{p})_o$
C_{y_r}	$(\partial C_y/\partial \hat{r})_o$
C_{y_β}	$(\partial C_y/\partial \beta)_o$
$C_{y_{\delta a}}$	$(\partial C_y/\partial \delta a)_o$
$C_{y_{\delta r}}$	$(\partial C_y/\partial \delta r)_o$
C_z	$(Z/1/2\rho S V_c^2)_o$
C_{z_q}	$(\partial C_z/\partial \hat{q})_o$
C_{z_u}	$(\partial C_z/\partial \hat{u})_o$
C_{z_α}	$(\partial C_z/\partial \alpha)_o$
$C_{z_{\dot\alpha}}$	$[\partial C_z/\partial(\dot\alpha\bar{c}/2U_o)]_o$
$C_{z_{\delta e}}$	$(\partial C_z/\partial \delta e)_o$

INVESTIGATION OF STABILITY AND CONTROL CHARACTERISTICS OF AC130 LINEAR MODELS

I. Introduction

Background

The weapons and fire control system of the AC130 gunship posses a capability for extremely high accuracy, *if* the aircraft platform can be rigidly controlled within crucial tolerances. A novel automatic control system, termed the Sight Line Autopilot (SLAP) by its innovators, is presently under development. The system will be capable of maintaining the aircraft in its firing orbit within stringent limits, allowing the pilot to devote his attention to firing the weapons.

The replacement of the human pilot by this control system is conditional on that system's capability to anticipate accurately the aircraft's perturbed response to a disturbance from trim. A comprehensive mathematical model is, therefore, a necessity. In determining such a model, two sets of parameters are needed to describe adequately the dynamic idiosyncrasies of the airframe: stability derivatives and transfer functions. Stability derivatives, introduced into the perturbation equations, relate the changes in the

aerodynamic forces and moments with changes in the aircraft's state vectors when the aircraft is disturbed from trim. The state vectors in this study were: angle of attack, forward airspeed, sideslip angle, yaw angle, pitch angle, bank angle, and angular velocities. Transfer functions, determined primarily by the stability derivatives, are input/output ratios used within the autopilot to program the required changes back to the trim condition.

Purpose and Scope

The objectives of this study were twofold. First, more comprehensive, linear models of both the AC130A and AC130E aircraft were developed. The dynamic characteristics of the proposed AC130E model were then compared with those of an existing model. Second, data were generated for the AC130A throughout an extensive flight envelope to establish general trends and probable values of stability derivatives and selected stability and control parameters. This data will be used to compare model response with flight test results on the AC130A aircraft. Aeroelastic effects were not included in the model as no test data were available and no reliable technique has been developed for estimating these phenomena. Gust inputs and quantitative determination of transfer functions were not treated in the study as the author feels that, until a valid model has been confirmed, such efforts would not produce useful results. Because of the large number of calculations involved, the iterative

procedures, the dynamic response modeling, and the plotting of data were all programmed in Extended Fortran language for the CDC 6600 digital computer and the Calcomp package.

II. Governing Equations

The equations used in the mathematical model were derived from Etkin (Ref 6), and include the gyroscopic coupling terms. Assuming the airframe to be a rigid body and the earth axes to be fixed in space, the complete set of equations is:

Equations of Motion

$$X+X_T-mg\sin\Theta = m(\dot{U}+QW-RV) \tag{1}$$

$$Z+Z_T+mg\cos\Theta\cos\Phi = m(\dot{W}+PV-QU) \tag{2}$$

$$\mathcal{M}+\mathcal{M}_T = \dot{Q}I_y+PR(I_x-I_z)+(QR+\dot{P})I_{xy}+(PQ-\dot{R})I_{yz}$$

$$+(P^2-R^2)I_{xz}+h_xR-h_zP \tag{3}$$

$$Y+Y_T+mg\cos\Theta\sin\Phi = m(\dot{V}+RU-PW) \tag{4}$$

$$\mathcal{L}+\mathcal{L}_T = \dot{P}I_x+QR(I_z-I_y)-(PQ-\dot{R})I_{xz}+(PR-\dot{Q})I_{xy}$$

$$+(R^2-Q^2)I_{yz}-h_yR+h_zQ \tag{5}$$

$$\mathcal{N}+\mathcal{N}_T = \dot{R}I_z+PQ(I_y-I_x)+(QR-\dot{P})I_{xz}+(PR-\dot{Q})I_{yz}$$

$$+(Q^2-P^2)I_{xy}-h_xQ+h_yP \tag{6}$$

Euler Relationships

$$P = \dot{\Theta} - \dot{\Psi}\sin\Theta \qquad (7)$$

$$Q = \dot{\Theta}\cos\Phi + \dot{\Psi}\cos\Theta\sin\Phi \qquad (8)$$

$$R = -\dot{\Theta}\sin\Phi + \dot{\Psi}\cos\Theta\cos\Phi \qquad (9)$$

$$\dot{\Theta} = Q\cos\Phi - R\sin\Phi \qquad (10)$$

$$\dot{\Phi} = (Q\sin\Phi + R\cos\Phi)\tan\Theta + P \qquad (11)$$

$$\dot{\Psi} = (Q\sin\Phi + R\cos\Phi)\sec\Theta \qquad (12)$$

Inertial Velocity

$$\dot{z}_i = -U\sin\Theta + V\cos\Theta\sin\Phi + W\cos\Theta\cos\Phi \qquad (13)$$

To simplify these equations further, the following assumptions and restrictions were made:

1. The only significant rotors contributing to gyroscopic oupling were the propellers. $\rightarrow h_y = h_z = 0$, $h_x = \Omega I$.

2. The xz plane was a plane of symmetry. $\rightarrow I_{xy} = I_{yz} = 0$, $Y_T = \mathcal{L}_T = \mathcal{N}_T = 0$.

3. The thrust force was parallel to the longitudinal axis. $\rightarrow Z_T = 0$.

4. The angles of attack and sideslip were not allowed to exceed ten degrees. The following linear approximations were then used: $U \doteq V_c$, $\alpha \doteq W/U$, $\beta \doteq V/U$, $\dot{W} \doteq U\dot{\alpha} + \dot{U}\alpha$, $\dot{V} \doteq U\dot{\beta} + \dot{U}\beta$.

Employing these assumptions, the equations became:

$$X+X_T-mg\sin\Theta = m(\dot{U}+UQ\alpha-UR\beta) \tag{1a}$$

$$Z+mg\cos\Theta\cos\Phi = m(\dot{U}\alpha+U\dot{\alpha}+UP\beta-UQ) \tag{2a}$$

$$\mathcal{M}+\mathcal{M}_T = \dot{Q}I_y+PR(I_x-I_z)+(P^2-R^2)I_{xz}+\Omega IR \tag{3a}$$

$$Y+mg\cos\Theta\sin\Phi = m(\dot{U}\beta+U\dot{\beta}+UR-UP\alpha) \tag{4a}$$

$$\mathcal{L} = \dot{P}I_x+QR(I_z-I_y)-(PQ+\dot{R})I_{xz} \tag{5a}$$

$$\mathcal{N} = \dot{R}I_z+PQ(I_y-I_x)+(QR-\dot{P})I_{xz}-\Omega IQ \tag{6a}$$

Equations (7) through (12) were unchanged.

$$\dot{z}_i = U(-\sin\Theta+\beta\cos\Theta\sin\Phi+\alpha\cos\Theta\cos\Phi). \tag{13a}$$

Solving the equations for time-dependent parameters, the following set of equations referred to the trim conditions were obtained:

$$2m\dot{U}_o = \rho_o S U_o^2 C_x + 2T_o - 2mg\sin\theta_o + 2mU_o(R_o\beta_o - Q_o\alpha_o) \tag{14}$$

$$2mU_o\dot{\alpha}_o = \rho_o S U_o^2(C_z - \alpha_o C_x) - 2\alpha_o T_o + 2mg(\cos\theta_o\cos\phi_o+\alpha_o\sin\theta_o)$$
$$+ 2mU_oQ_o(1+\alpha_o^2) - 2mU_o\beta_o(P_o+R_o\alpha_o) \tag{15}$$

$$2I_y\dot{Q}_o = \rho_o S U_o^2 \bar{c} C_m + 2P_oR_o(I_z-I_x) + 2I_{xz}(R_o^2-P_o^2) + 2I_y\Omega IR_o \tag{16}$$

$$\dot{\theta}_o = Q_o\cos\phi_o - R_o\sin\phi_o \tag{17}$$

$$2mU_o\dot{\beta}_o = \rho_o SU_o^2(C_y-\beta_o C_x) - 2\beta_o T_o + 2mg(\cos\theta_o\sin\phi_o+\beta_o\sin\theta_o)$$

$$- 2mU_oR_o(1+\beta_o^2) + 2mU_o\alpha_o(P_o+Q_o\beta_o) \qquad (18)$$

$$2(I_xI_z-I_{xz}^2)\dot{P}_o = \rho_o SU_o^2 b(I_zC_l+I_{xz}C_n) + 2Q_oR_o(I_yI_z-I_z^2+I_{xz}^2)$$

$$+ 2P_oQ_oI_{xz}(I_z+I_x-I_y) + 2Q_oI_{xz}\Omega I \qquad (19)$$

$$2(I_xI_z-I_{xz}^2)\dot{R}_o = \rho_o SU_o^2 b(I_xC_n+I_{xz}C_l) + 2Q_oP_o(I_x^2-I_xI_y+I_{xz}^2)$$

$$+ 2R_oQ_oI_{xz}(I_y-I_z-I_x) + 2Q_oI_x\Omega I \qquad (20)$$

$$\dot{\phi}_o = P_o + (Q_o\sin\phi_o+R_o\cos\phi_o)\tan\theta_o \qquad (21)$$

$$\dot{\psi}_o = (Q_o\sin\phi_o+R_o\cos\phi_o)\sec\theta_o \qquad (22)$$

$$\dot{z}_{i_o} = U_o(-\sin\theta_o+\beta_o\cos\theta_o\sin\phi_o+\alpha_o\cos\theta_o\cos\phi_o) \qquad (23)$$

Thrust appears explicitly in equations (14), (15), and (18), while in equation (16) this term is absorbed into the coefficient C_m. This arrangement was to facilitate an iteration process used to obtain the initial (trim) conditions.

Perturbations about the trim condition were then introduced. These disturbances were considered small compared to the steady-state values of the state vectors. The sines and cosines of the perturbation angles were approximated by the angles themselves and one (1), respectively. The products and squares of the perturbation quantities were considered negligible in comparison to the perturbations themselves. The following notation is used in this report: $U = U_o + u$, $\Phi = \phi_o + \phi$, etc, where the subscript denotes the

state variable at its trim condition and the lower case character denotes a perturbation. Assuming that inertia terms ($\dot{I}_x$, etc) and atmospheric variables (density, speed of sound, etc) remain essentially constant for the small time span under consideration, the following perturbation equations were obtained (direct thrust effects here were included in the aerodynamic forces):

$$\Delta X/m = \dot{u} + (\alpha_o Q_o + R_o \beta_o)u + U_o Q_o \alpha + U_o \alpha_o q + g\theta\cos\theta_o - U_o R_o \beta - U_o \beta_o r \qquad (24)$$

$$\Delta Z/m = \alpha_o \dot{u} + U_o \dot{\alpha} + (\dot{\alpha}_o - Q_o)u + \dot{U}_o \alpha - U_o q + g\theta\sin\theta_o\cos\phi_o + P_o U_o \beta + U_o \beta_o p + g\phi\cos\theta_o\sin\phi_o \qquad (25)$$

$$\Delta \mathfrak{M} = I_y \dot{q} + (I_x R_o - I_z R_o + 2I_{xz} P_o)p + (I_x P_o - I_z P_o - 2I_{xz} R_o + \Omega I)r \qquad (26)$$

$$\Delta \dot{\theta} = q\cos\theta_o - r\sin\phi_o - \phi\dot{\psi}_o\cos\theta_o \qquad (27)$$

$$\Delta Y/m = \beta_o \dot{u} + U_o \dot{\beta} + (R_o - P_o \alpha_o + \dot{\beta}_o)u - P_o U_o \alpha + g\theta\sin\theta_o\sin\phi_o + \dot{U}_o \beta - \alpha_o U_o p + U_o r - g\phi\cos\theta_o\cos\phi_o \qquad (28)$$

$$\Delta \mathcal{L} = I_x \dot{p} - I_{xz} \dot{r} + (I_z R_o - I_y R_o - I_{xz} P_o)q + Q_o(I_z - I_y)r - I_{xz} Q_o p \qquad (29)$$

$$\Delta \mathfrak{N} = I_z \dot{r} - I_{xz} \dot{p} + (I_y P_o - I_x P_o + I_{xz} R_o - \Omega I)q + Q_o(I_y - I_x)p + I_{xz} Q_o r \qquad (30)$$

$$\Delta \dot{\phi} = q\tan\theta_o\sin\phi_o + (\dot{\phi}_o\tan\theta_o - P_o\tan\theta_o + Q_o\sin\phi_o + R_o\cos\phi_o)\theta + p + (\tan\theta_o\cos\phi_o)r + (\dot{\theta}_o\tan\theta_o)\phi \qquad (31)$$

$$\Delta\dot{\psi} = (q\sin\phi_o + \theta Q_o \tan\theta_o \sin\phi_o + \theta R_o \tan\theta_o \cos\phi_o + r\cos\phi_o)\sec\theta_o + \phi\dot{\theta}_o \sec\theta_o \quad (32)$$

Equations (24) through (32) were then expanded by linearizing the aerodynamic forces and moments and expressing them in terms of the non-dimensional stability derivatives. The following assumptions were applied to the linearization procedure:

1. Cross derivatives (C_{x_y}, C_{n_u}, etc) were considered negligible (Ref 6:123-4).

2. The thrust coefficient was independent of angle of attack (Ref 6:147).

3. The change in side force due to aileron deflection ($C_{y_{\delta a}}$) was negligible (Ref 3:88).

4. The airflow around the aircraft was assumed to be quasi-steady. Unsteady flow effects were considered negligible and all derivatives with respect to accelerations were omitted with the exception of those due to $\dot{\alpha}$. The $\dot{\alpha}$ derivatives were retained to account for the effects of downwash on the horizontal tail (Ref 9:4-52, Ref 4:370).

The linearized aerodynamic forces and moments, adapted from Ref 6, were used in the following form:

$$\Delta X = \tfrac{1}{2}\rho_o S U_o (2C_x + C_{x_u})u + \tfrac{1}{2}\rho_o S U_o^2 (C_{x_\alpha}\alpha + \frac{\bar{c}}{2U_o} C_{x_q} q + C_{x_{\delta e}} \delta e)$$

$$\Delta Z = \tfrac{1}{2}\rho_o S U_o (2C_z + C_{z_u})u + \tfrac{1}{2}\rho_o S U_o^2 (C_{z_\alpha}\alpha + \frac{\bar{c}}{2U_o} C_{z_{\dot{\alpha}}}\dot{\alpha} + \frac{\bar{c}}{2U_o} C_{z_q} q + C_{z_{\delta e}} \delta e)$$

$$\Delta\mathcal{M} = \frac{1}{2}\rho_o S U_o \bar{c}(2C_m + C_{m_u})u + \frac{1}{2}\rho_o S U_o^2 \bar{c}(C_{m_\alpha}\alpha + \frac{\bar{c}}{2U_o}C_{m_{\dot{\alpha}}}\dot{\alpha} + \frac{\bar{c}}{2U_o}C_{m_q}q + C_{m_{\delta e}}\delta e)$$

$$\Delta Y = \frac{1}{2}\rho_o S U_o(2C_y)u + \frac{1}{2}\rho_o S U_o^2(C_{y_\beta}\beta + \frac{b}{2U_o}C_{y_p}p + \frac{b}{2U_o}C_{y_r}r + C_{y_{\delta r}}\delta r)$$

$$\Delta\mathcal{L} = \frac{1}{2}\rho_o S U_o b(2C_l)u + \frac{1}{2}\rho_o S U_o^2 b(C_{l_\beta}\beta + \frac{\bar{c}}{2U_o}C_{l_p}p + \frac{\bar{c}}{2U_o}C_{l_r}r + C_{l_{\delta a}}\delta a + C_{l_{\delta r}}\delta r)$$

$$\Delta\mathcal{N} = \frac{1}{2}\rho_o S U_o b(2C_n)u + \frac{1}{2}\rho_o S U_o^2 b(C_{n_\beta}\beta + \frac{\bar{c}}{2U_o}C_{n_p}p + \frac{\bar{c}}{2U_o}C_{n_r}r + C_{n_{\delta a}}\delta a + C_{n_{\delta r}}\delta r)$$

III. Development of Model

Equations

The linearized forces and moments were substituted into the perturbation equations (24) through (32) and the resulting set of equations solved for the time derivatives of the state variables. The steady-state (trim) condition of level, turning flight was simulated by requiring the following restrictions:

1. All state vectors were constants except ψ_o, which was cyclic, and β_o, which was zero.
2. Accelerations of state variables were zero.

Applying these conditions to the perturbation equations, the following set, referred to body axes, resulted:

$$\dot{u} = [\frac{\rho_o S U_o}{2m}(2C_x + C_{x_u}) - \alpha_o Q_o]u + [\frac{\rho_o S U_o^2}{2m}C_x - U_o Q_o]\alpha$$

$$+ [\frac{\rho_o S U_o \bar{c}}{4m}C_{x_{\dot{\alpha}}}]\dot{\alpha} + [\frac{\rho_o S U_o \bar{c}}{4m}C_{x_q} - \alpha_o U_o]q$$

$$- [g\cos\theta_o]\theta + [U_o R_o]\beta + [\frac{\rho_o S U_o^2}{2m}C_{x_{\delta e}}]\delta e \qquad (33)$$

$$[1-\frac{\rho_o S\bar{c}}{4m}C_{z_{\dot{\alpha}}}]\dot{\alpha} = [\frac{\rho_o S}{2m}(2C_z+C_{z_u})+\frac{Q_o}{U_o}]u - [\frac{\alpha o}{U_o}]\dot{u} + [\frac{\rho_o SU_o}{2m}C_{z_\alpha}]\alpha$$

$$+ [1 + \frac{\rho_o S\bar{c}}{4m}C_{z_q}]q - [\frac{g}{U_o}\sin\theta_o\cos\phi_o]\theta$$

$$- [P_o]\beta - [\frac{g}{U_o}\cos\theta_o\sin\phi_o]\phi + [\frac{\rho_o SU_o}{2m}C_{z_{\delta e}}]\delta e \quad (34)$$

$$I_y\dot{q} = [\frac{\rho_o SU_o\bar{c}}{2}(2C_m-C_{m_u})]u + [\frac{\rho_o SU_o^2\bar{c}}{2}C_{m_\alpha}]\alpha + [\frac{\rho_o SU_o\bar{c}^2}{4}C_{m_{\dot{\alpha}}}]\dot{\alpha}$$

$$+ [\frac{\rho_o SU_o\bar{c}^2}{4}C_{m_q}]q + (I_zR_o-I_xR_o-2I_{xz}P_o)p$$

$$+ [\frac{\rho_o SU_o^2\bar{c}}{2}C_{m_{\delta e}}]\delta e + (I_zP_o-I_xP_o+2I_{xz}R_o-\Omega I)r \quad (35)$$

$$\dot{\theta} = (\cos\phi_o)q - (\sin\phi_o)r - (\dot{\psi}_o\cos\theta_o)\phi \quad (36)$$

$$\dot{\beta} = [\frac{\rho_o S}{m}C_y+\frac{\alpha_o P_o-R_o}{U_o}]u + (P_o)\alpha - [\frac{g}{U_o}\sin\theta_o\sin\phi_o]\theta$$

$$+ [\frac{\rho_o SU_o}{2m}C_{y_\beta}]\beta + [\frac{\rho_o Sb}{4m}C_{y_p}+\alpha_o]p + [\frac{\rho_o Sb}{4m}C_{y_r}-1]r$$

$$+ [\frac{g}{U_o}\cos\theta_o\cos\phi_o]\phi + [\frac{\rho_o SU_o}{2m}C_{y_{\delta r}}]\delta r \quad (37)$$

$$[\frac{I_x}{I_{xz}}-\frac{I_{xz}}{I_z}]\dot{p} = \rho_o SU_o b[\frac{C_l}{I_{xz}} + \frac{C_n}{I_z}]u + \frac{\rho_o SU_o^2 b}{2}[\frac{C_{l_\beta}}{I_{xz}} + \frac{C_{n_\beta}}{I_z}]\beta$$

$$+ [(\frac{I_x-I_y}{I_z}+1)P_o + (\frac{I_y-I_z}{I_{xz}} - \frac{I_{xz}}{I_z})R_o + \frac{I}{I_z}]q$$

$$+ [\frac{\rho_o SU_o b^2}{4}(\frac{C_{lp}}{I_{xz}}+\frac{C_{np}}{I_z})+ Q_o(1+\frac{I_x-I_y}{I_z})]p$$

$$+\left[\frac{\rho_o S U_o b^2}{4}\left(\frac{C_{l_r}}{I_{xz}} + \frac{C_{n_r}}{I_z}\right) + Q_o\left(\frac{I_y - I_z}{I_{xz}} - \frac{I_{xz}}{I_z}\right)\right]r$$

$$+\frac{\rho_o S U_o^2 b}{2}\left[\frac{C_{l_{\delta a}}}{I_{xz}} + \frac{C_{n_{\delta a}}}{I_z}\right]\delta a + \frac{\rho_o S U_o^2 b}{2}\left[\frac{C_{l_{\delta r}}}{I_{xz}} + \frac{C_{n_{\delta r}}}{I_z}\right]\delta r \tag{38}$$

$$\left[\frac{I_z}{I_{xz}} - \frac{I_{xz}}{I_x}\right]\dot{r} = \rho_o S U_o b\left[\frac{C_n}{I_{xz}} + \frac{C_l}{I_x}\right]u + \frac{\rho_o S U_o^2 b}{2}\left[\frac{C_{n_\beta}}{I_{xz}} + \frac{C_{l_\beta}}{I_x}\right]\beta$$

$$+\left[\left(\frac{I_y - I_z}{I_x} - 1\right)R_o + \left(\frac{I_x - I_y}{I_{xz}} + \frac{I_{xz}}{I_x}\right)P_o + \frac{I}{I_{xz}}\right]q$$

$$+\left[\frac{\rho_o S U_o b^2}{4}\left(\frac{C_{n_p}}{I_{xz}} + \frac{C_{n_p}}{I_x}\right) + Q_o\left(\frac{I_x - I_y}{I_{xz}} + \frac{I_{xz}}{I_x}\right)\right]p$$

$$+\left[\frac{\rho_o S U_o b^2}{4}\left(\frac{C_{n_r}}{I_{xz}} + \frac{C_{l_r}}{I_x}\right) + Q_o\left(\frac{I_y - I_z}{I_x} - 1\right)\right]r$$

$$+\frac{\rho_o S U_o^2 b}{2}\left[\frac{C_{n_{\delta a}}}{I_{xz}} + \frac{C_{l_{\delta a}}}{I_x}\right]\delta a + \frac{\rho_o S U_o^2 b}{2}\left[\frac{C_{n_{\delta r}}}{I_{xz}} + \frac{C_{l_{\delta r}}}{I_x}\right]\delta r \tag{39}$$

$$\dot{\phi} = (\tan\theta_o \sin\phi_o)q + (\dot{\psi}_o \cos\theta_o - P_o \tan\theta_o)\theta$$

$$+ (\tan\theta_o \cos\phi_o)r + p \tag{40}$$

$$\dot{\psi} = \left(\frac{\sin\phi_o}{\cos\theta_o}\right)q + (\dot{\psi}_o \tan\theta_o)\,\theta + \left(\frac{\cos\phi_o}{\cos\theta_o}\right)r \tag{41}$$

As can be seen, the numerical evaluation of the coefficient terms in these equations requires determination of the trim conditions and estimation of the application stability derivatives.

Initial (Trim) Conditions

The trim equations, (14) through (23), were used to compute initial conditions through iteration on α_o, T_o, C_l, C_m, and C_n. The iteration procedure was developed by personnel in the Department of Astronautics and Computer Science, U.S. Air Force Academy, and was expanded slightly for use in this study. The basis of the iteration was the requirement for constant state vectors at the trim condition. A listing of the computer program can be found in Appendix B.

Applying the restrictions listed on page 11 for level, steady, turning flight, the following equations were obtained:

$$\dot{U}_o = \frac{\rho_o S U_o^2}{2m} C_x + \frac{T_o}{m} - g\sin\theta_o - U_o Q_o \alpha_o \tag{42}$$

$$\dot{\alpha}_o = \frac{\rho_o S U_o}{2m} (C_z - \alpha_o C_x) - \frac{\alpha_o T_o}{m U_o} + \frac{g}{U_o}(\cos\theta_o \cos\phi_o + \alpha_o \sin\theta_o) + Q_o(1+\alpha_o^2) \tag{43}$$

$$P_o = -\dot{\psi}_o \sin\theta_o \tag{44}$$

$$Q_o = \dot{\psi}_o \cos\theta_o \sin\phi_o \tag{45}$$

$$R_o = \dot{\psi}_o \cos\theta_o \cos\phi_o \tag{46}$$

$$\alpha_o = \tan\theta_o / \cos\phi_o \tag{47}$$

From Ref 8:203, the following relationships were obtained for the reference flight condition:

$$R = \frac{V_c^2}{g\tan\phi_o} \tag{48}$$

$$\dot{\psi}_o = \frac{V_c}{R} \tag{49}$$

Using data from Ref 1, approximate values of α_o and T_o were computed to begin the iterative process.

Iteration was first performed on equations (42) and (43). It was assumed that, for small disturbances,

$$\left[\Delta\dot{u}\right] = \left[\frac{\partial\dot{u}}{\partial\alpha}\Delta\alpha + \frac{\partial\dot{u}}{\partial T}\right]\left[\Delta T\right] \tag{50}$$

$$\left[\Delta\dot{\alpha}\right] = \left[\frac{\partial\dot{\alpha}}{\partial\alpha}\Delta\alpha + \frac{\partial\dot{\alpha}}{\partial T}\right]\left[\Delta T\right] \tag{51}$$

The following equations were then solved for $\Delta\alpha$ and ΔT by Cramer's Rule.

$$\begin{bmatrix} \Delta\dot{u} \\ \\ \Delta\dot{\alpha} \end{bmatrix} = \begin{bmatrix} \frac{\partial\dot{u}}{\partial\alpha} & \frac{\partial\dot{u}}{\partial T} \\ \\ \frac{\partial\dot{\alpha}}{\partial\alpha} & \frac{\partial\dot{\alpha}}{\partial T} \end{bmatrix} \begin{bmatrix} \Delta\alpha \\ \\ \Delta T \end{bmatrix}$$

where $\Delta\dot{u} = \dot{U}_o - \dot{u}$ computed and $\Delta\dot{\alpha} = \dot{\alpha}_o - \dot{\alpha}$ computed. Final values for α_o and T_o were obtained by setting $\dot{U}_o = \dot{\alpha}_o = 0$ and noting that requiring $\Delta\alpha = \Delta T = 0$ will result in $\dot{u}$ computed $= \dot{\alpha}$ computed $= 0$.

Values were thus obtained for α_o, θ_o, P_o, Q_o, and R_o. C_x and C_z were determined by summing forces along the respective axes, giving

$$C_x = C_T + C_L \sin\alpha_o - C_D \cos\alpha_o \tag{52}$$

$$C_z = -C_L \cos\alpha_o - C_D \sin\alpha_o \tag{53}$$

Trim conditions were then impressed upon equations (16), (18), (19), and (20) and these equations solved for C_m, C_y, C_l, and C_n. A second iterative procedure was performed using equations (16), (19), and (20). The matrix which was used to solve for final values of C_l, C_m, and C_n was

$$\begin{bmatrix} \Delta\dot{p} \\ \Delta\dot{q} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \frac{\partial\dot{p}}{\partial C_l} & \frac{\partial\dot{p}}{\partial C_m} & \frac{\partial\dot{p}}{\partial C_n} \\ \frac{\partial\dot{q}}{\partial C_l} & \frac{\partial\dot{q}}{\partial C_m} & \frac{\partial\dot{q}}{\partial C_n} \\ \frac{\partial\dot{r}}{\partial C_l} & \frac{\partial\dot{r}}{\partial C_m} & \frac{\partial\dot{r}}{\partial C_n} \end{bmatrix} \begin{bmatrix} \Delta C_l \\ \Delta C_m \\ \Delta C_n \end{bmatrix}$$

where $\Delta\dot{p} = \dot{P}_o - \dot{p}$ computed, $\Delta\dot{q} = \dot{Q}_o - \dot{q}$ computed, and $\Delta\dot{r} = \dot{R}_o - \dot{r}$ computed.

Stability Derivatives

Formulas used in estimating stability derivatives are presented in Table I.

Table 1. Stability Derivatives

	Formula	Source	Restrictions
C_{x_u}	$C_{T_u}+\alpha_o(C_{L_u}-C_L)-\alpha_o^2 C_D-C_{D_u}$	Ref 6:148-52	$C_{D_M}=0$, $T(U_o+u)$-const, Neglect Aeroelastic Effects
C_{z_u}	$\alpha_o(C_D-C_{D_u})-C_{L_u}-\alpha_o^2 C_L$	Ref 1:35	$C_{L_u}=\frac{M^2 C_L}{1-M^2}-\alpha C_{L_\alpha}$, $C_{D_u}=-\alpha C_{D_\alpha}$ Body Axis System
C_{m_u}	$C_{L_u}(\frac{\partial C_m}{\partial C_L})_o$	Ref 6:151-2	Neglect Aeroelastic Effects $-0.1 \le C_m \le 0.1$
		Ref 1:39,41	Stability Axis System
C_{x_α}	$C_L-C_{D_\alpha}+\alpha_o(C_D+C_{L_\alpha})$	Ref 1:39,41	Body Axis System
C_{z_α}	$\alpha_o(C_L-C_{D_\alpha})-C_D-C_{L_\alpha}$		
C_{m_α}	$C_{L_\alpha}(\frac{\partial C_m}{\partial C_L})c$	Ref 8:392	Stability Axis System, $T_c \le 0.2$, $-0.1 \le C_m \le 0.1$
$C_{x_{\dot\alpha}}$	Negligible	Ref 6:158-65	10% Added for Wing-Body Contribution
$C_{z_{\dot\alpha}}$	$-2.2\, n_t \frac{l_t}{\bar c}\frac{\partial\varepsilon}{\partial\alpha}$	Ref 4:369	
$C_{m_{\dot\alpha}}$	$-2.2\, n_t (\frac{l_t}{\bar c})^2 \frac{\partial\varepsilon}{\partial\alpha}$	Ref 3:54,55	Stability Axis System

Table 1. (continued)

C_{x_q}	Negligible	Ref 4:376	
C_{z_q}	$-2.2 \frac{l_t}{\bar{c}} \eta_t$	Ref 6:153-55	Stability Axis System
C_{m_q}	$-2.2 (\frac{l_t}{\bar{c}})^2 \eta_t$	Ref 3:54,55	
$C_{x_{\delta e}}$	$-.080$	Ref 3:40	$\delta_e \pm 10$ degrees
$C_{z_{\delta e}}$	$-\frac{\bar{c}}{l_t} \eta_t C_{m_{\delta e}}$	Ref 3:44	Stability Axis System
$C_{m_{\delta e}}$	-1.644	Ref 1:68-9	
C_{y_β}	$-.888+.345F-(1.72-2.91F)T_c$ for $M \geq .225$, $C_{y_\beta}=C_{y_\beta}(1+.15(M-.225))$	Ref 1:78,79,81	
C_{l_β}	$-.0544+.0115F+(-.0014+.0029F)\alpha_o$, α_o in degrees	Ref 1:94-7	$\beta \pm 8$ degrees
C_{n_β}	$.0785+.0344F-.0859T_c$ for $M \geq .225$, $C_{n_\beta}=C_{n_\beta}(1+.2(M-.225))$	Ref 1:84-7	Stability Axis System

Table 1. (continued)

$C_{y_{\beta t}}$	$-.645+.0650F+(1.47-2.81F)T_c-(1.75-2.03F)T_c^2$		
	for $M>.225, C_{y_{\beta t}}=C_{y_{\beta t}}(1+.2(M-.225)$	Ref 3:94,119	
$C_{l_{\beta t}}$	$-.0638-.0034F-(.023+.090F)T_c-(.090+.500F)T_c^2$		-4 degrees $\leq T_c \leq 8$ degrees
		Ref 3:120	$T_c \leq 0.2$
$C_{n_{\beta t}}$	$.218-.0343F+(.745F-.430)T_c$	Ref 3:100-1	Stability Axis System
	for $M>.225$, $C_{n_{\beta t}}=C_{n_{\beta t}}(1+.4(M-.225))$	Ref 1:105	
C_{y_p}	$-.0167C_{L_w}-2\alpha_o(\frac{l_f}{b})C_{y_{\beta t}}$	Ref 5:7.4.2.1	
C_{l_p}	$-.530$ $M < .225$ $-.530-.114(M-.225)$, $M \geq .225$	Ref 1:99	Stability Axis System
C_{n_p}	$-.0015-.09C_{L_w}-2(\frac{l_f}{b})\alpha_o C_{n_{\beta t}}$	Ref 5:7.4.2.3	
C_{y_r}	$2\ C_{n_{\beta t}}$	Ref 5:7.4.3.1	
C_{l_r}	$.29[1+\frac{10.09M^2}{20.18(1-M^2)+4\sqrt{1-M^2}}]C_{L_w}+.0518F$ $-C_{l_\beta}-.0167C_{y_r}+2[\frac{C_{l_{\beta_t}}C_{n_{\beta t}}}{C_{y_{\beta t}}}]$	Ref 5.7.4.3.2	Stability Axis System

Table 1. (continued)

C_{n_r}	$=.02C_{L_w}^2-.3C_{D_{o_w}}+2[\frac{C_{n_{\beta_t}}^2}{C_{y_{\beta_t}}}]$	Ref 5:7.4.3.3	Stability Axis System
$C_{y_{\delta_a}}$	negligible	Ref 3:88	
$C_{l_{\delta_a}}$	$-.0602-.0170F+.0290(C_L-.8)$ $,0.8\leq C_L<1.0$		
	$-.0544-.0172F+(.0140+.03F)(C_L-1.0),1.0\leq C_L<1.2$		δ_a(total)$\pm$15 degrees
	$-.0516-.0114F+.0290(C_L-1.2)$ $,1.2\leq C_L<1.6$		Stability Axis System
		Ref 1:105-6	
$C_{n_{\delta_a}}$	$.00332+.00115\alpha_o$,0degrees $\leq \alpha_o<2$ degrees		
	$.00556+.00072(\alpha_o-2)$,2degrees $\leq \alpha_o<8$ degrees		
		Ref 1:108	
$C_{y_{\delta_r}}$	0.264		
$C_{l_{\delta_r}}$	$-.0264-.0015\alpha_o,\alpha_o$ in degrees	Ref 1:88-90	$\delta_r\pm$10 degrees
			Stability Axis System
$C_{n_{\delta_r}}$	$-.0917$		

Because the model was to be referred to the body axes, the following transformations (Ref 10:57-8) were then applied to those stability derivatives which were originally referenced to the stability axes.

$$C_{m_{u_B}} = C_{m_u} - \alpha_o C_{m_\alpha} \qquad C_{l_{\beta_B}} = C_l - \alpha_o C_n$$

$$C_{m_{\alpha_B}} = C_{m_\alpha} + \alpha_o C_{m_u} \qquad C_{n_{\beta_B}} = C_n + \alpha_o C_l$$

$$C_{x_{q_B}} = C_{x_q} - \alpha_o C_{z_q} \qquad C_{l_{p_B}} = C_{l_p} - \alpha_o (C_{l_r} + C_{n_p}) + \alpha_o^2 C_{n_r}$$

$$C_{z_{q_B}} = C_{z_q} + \alpha_o C_{x_q} \qquad C_{n_{p_B}} = C_{n_p} + \alpha_o (C_{l_p} - C_{n_r}) - \alpha_o^2 C_{l_r}$$

$$C_{x_{\dot{\alpha}_B}} = C_{x_{\dot{\alpha}}} - \alpha_o C_{z_{\dot{\alpha}}} \qquad C_{l_{r_B}} = C_{l_r} + \alpha_o (C_{l_p} - C_{n_r}) - \alpha_o^2 C_{n_p}$$

$$C_{z_{\dot{\alpha}_B}} = C_{z_{\dot{\alpha}}} + \alpha_o C_{x_{\dot{\alpha}}} \qquad C_{n_{r_B}} = C_{n_r} + \alpha_o (C_{l_r} + C_{n_p}) + \alpha_o^2 C_{l_p}$$

$$C_{x_{\delta e_B}} = C_{x_{\delta e}} - \alpha_o C_{z_{\delta e}} \qquad C_{l_{\delta a_B}} = C_{l_{\delta a}} - \alpha_o C_{n_{\delta a}}$$

$$C_{z_{\delta e_B}} = C_{z_{\delta e}} + \alpha_o C_{x_{\delta e}} \qquad C_{n_{\delta a_B}} = C_{n_{\delta a}} + \alpha_o C_{l_{\delta a}}$$

$$C_{y_{p_B}} = C_{y_p} - \alpha_o C_{y_r} \qquad C_{l_{\delta r_B}} = C_{l_{\delta r}} - \alpha_o C_{n_{\delta r}}$$

$$C_{y_{r_B}} = C_{y_r} + \alpha_o C_{y_p} \qquad C_{n_{\delta r_B}} = C_{n_{\delta r}} + \alpha_o C_{l_{\delta r}}$$

All other derivatives were either unchanged or were initially defined in the body axis system.

Model Characteristics

After initial conditions and stability derivatives had been evaluated, equations (33) through (41) were arranged in matrix form

$$\dot{X} = A X$$

where $\dot{X}$ was the 9x1 matrix of time derivatives ($\dot{u}$, $\dot{\alpha}$, $\dot{q}$, $\dot{\theta}$, $\dot{\beta}$, $\dot{p}$, $\dot{r}$, $\dot{\phi}$, $\dot{\psi}$), X was the 12x1 matrix of state vectors and control terms (u, α, q, θ, β, p, r, ϕ, ψ, δa, δe, δr), and A was the 9x12 coefficient matrix determined by trim conditions and the stability derivatives.

Mode parameters were estimated by factoring the coefficient matrix A, using an eigenvalue program from Ref 11. The procedure yielded one zero root, two real roots, and three complex pairs. Factoring the complex roots gave the natural (undamped) frequencies and damping ratios for the Phugoid, Short Period, and Dutch Roll modes. The real roots determined the time constants for the Rolling and Spiral modes.

The perturbation equations were then integrated using a fourth-order Runge-Kutta routine. The subroutines were arranged so that the perturbed response of the state vectors could be estimated from a single control deflection or any combination of aileron, elevator, and rudder deflections. The computer programs used to determine the above characteristics are listed in Appendix D.

C130 A and E Model Differences

Only two parameters differ significantly between the two aircraft, both due to the fact that different propellers are used. The propeller moment of inertia (I) and the specific thrust (T_c) for each aircraft are listed in Appendix A.

IV. Results

Comparison of AC130E Models

Response to a one-second pulse of 0.1 radian was investigated for each of the primary control surfaces. Perturbations produced by the model developed in this study were compared to those produced by an existing model. The trim condition was turning flight at a constant altitude of 10,500 feet, at 28 degrees of bank, at a gross weight of 110,000 lb, and with C.G. at 25% MAC. Model parameters are listed in Tables II and III, and results of the computer simulations are presented in Appendix C.

A modified sensitivity analysis was performed by equating pairs of parameters which differed considerably and noting any subsequent change in response. Table IV contains the results of this analysis.

Compared to the existing model, the proposed model showed an increase in damping of phugoid oscillations. The primary dutch roll oscillations of the proposed model were generally weaker, but the β and r response to aileron deflection showed a marked decrease in dutch roll damping. Pronounced dutch roll was evident in several of the cross coupled perturbations of the proposed model, but the magnitudes of these oscillations were quite small. The divergent mode tended to be weaker in the proposed model. The short period oscillations of both models were identical.

Table II. Model Parameters - AC130E

	Proposed	Present		Proposed	Present		Proposed	Present
I	175.4	0.000	C_m	-0.00216	-0.00002	$C_{m_{\delta e}}$	-1.644	-1.634
I_x	1.4×10^6	same	C_n	-0.00012	-0.00000	C_{y_β}	-0.747	-0.745
I_y	9.7×10^5	same	C_{x_u}	-0.261	-0.145	C_{l_β}	-0.0464	-0.0461
I_z	2.2×10^6	same	C_{z_u}	0.154	-0.0757	C_{n_β}	0.0870	0.0819
I_{xz}	6.1×10^4	same	C_{m_u}	0.0354	-0.0167	C_{y_p}	-0.134	0.000
α_o	0.0307	0.0276	C_{x_α}	0.752	0.760	C_{l_p}	-0.542	-0.539
θ_o	0.0271	0.0244	C_{z_α}	-7.544	-7.343	C_{n_p}	-0.1630	-0.0391
ϕ_o	-0.489	same	C_{m_α}	-1.739	-1.600	C_{y_r}	0.386	0.404
$\dot{\psi}_o$	-0.0578	same	$C_{x_{\dot\alpha}}$	0.0982	0.0000	C_{l_r}	0.323	0.263
P_o	0.00156	0.00141	$C_{z_{\dot\alpha}}$	-3.203	-3.610	C_{n_r}	-0.174	-0.147
Q_o	0.0271	same	$C_{m_{\dot\alpha}}$	-10.263	-11.530	$C_{y_{\delta a}}$	0.000	0.000
R_o	-0.0510	same	C_{x_q}	0.240	0.000	$C_{l_{\delta a}}$	-0.0648	-0.0637
C_x	0.0289	0.0259	C_{z_q}	-7.822	-7.870	$C_{n_{\delta a}}$	0.0034	0.0000
C_y	-0.00037	-0.00015	C_{m_q}	-25.067	-25.100	$C_{y_{\delta r}}$	0.263	0.266
C_z	-0.943	same	$C_{x_{\delta e}}$	-0.0625	0.0000	$C_{l_{\delta r}}$	0.0265	0.0252
C_l	-0.0001	same	$C_{z_{\delta e}}$	-0.572	-1.125	$C_{n_{\delta r}}$	-0.0910	-0.0944

Table III

Mode Parameters - AC130E Models

Parameter	Present Model	Proposed Model
Phugoid Mode		
w_n (rad/sec)	0.146	0.144
	0.001	0.043
Short Period Mode		
w_n (rad/sec)	1.984	2.055
	0.636	0.606
Dutch Roll Mode		
w_n (rad/sec)	0.866	0.968
	0.215	0.253
Rolling Mode		
Time Constant (sec)	0.663	0.686
Spiral Mode		
Time Constant (sec)	69.31	66.09

Table IV. Perturbations of Proposed AC130E Model

Response	Comparison with Existing Model	Reason
$\frac{\alpha}{\delta a}$	Less dutch roll damping	I
$\frac{\theta}{\delta a}$	Less dutch roll damping	C_{n_p}
$\frac{u}{\delta a},\frac{\alpha}{\delta a},\frac{q}{\delta a},\frac{\theta}{\delta a}$	More phugoid damping	C_{x_u},C_{m_u},C_{n_p}
$\frac{\beta}{\delta a},\frac{r}{\delta a},\frac{\phi}{\delta a},\frac{\psi}{\delta a}$	Less dutch roll damping	C_{n_p}
$\frac{u}{\delta e},\frac{\theta}{\delta e}$	More phugoid damping	C_{x_u},C_{n_p}
$\frac{\beta}{\delta e},\frac{p}{\delta e},\frac{r}{\delta e}$	Less dutch roll damping	I
$\frac{\beta}{\delta e},\frac{r}{\delta e}$	More phugoid damping	$C_{x_u},C_{l_r},C_{n_r},C_{n_p}$
$\frac{\phi}{\delta e},\frac{\psi}{\delta e}$	More divergence	C_{l_r}
$\frac{\alpha}{\delta r},\frac{q}{\delta r}$	Less dutch roll damping	I
$\frac{u}{\delta r},\frac{\alpha}{\delta r},\frac{q}{\delta r},\frac{\theta}{\delta r}$	More phugoid damping	C_{m_u},C_{n_p}
$\frac{\theta}{\delta r},\frac{\beta}{\delta r},\frac{p}{\delta r},\frac{r}{\delta r},\frac{\phi}{\delta r},\frac{\psi}{\delta r}$	More dutch roll damping	C_{n_p}
$\frac{\phi}{\delta r},\frac{\psi}{\delta r}$	Less divergence	C_{m_u},C_{l_r},C_{n_p}

Mode Parameters and Stability Derivatives, AC130A

Selected mode parameters and stability derivatives were calculated for the AC130A aircraft in a level, left-hand turn at 30 degrees of bank, altitudes from 6000 feet to 15,000 feet in increments of 1000 feet, gross weights of 100,000 lb, 110,000 lb, and 120,000 lb, and C.G. locations of 15% MAC, 25% MAC (nominal), and 30% MAC. Results are presented graphically in Appendix D. From these data, probable ranges of the selected quantities were estimated to be

Phugoid Mode

w_n(rad/sec)--------------------0.115 to 0.145

ξ------------------------------0.025 to 0.050

Short Period Mode

w_n(rad/sec)--------------------2.200 to 2.700

ξ------------------------------0.550 to 0.650

Dutch Roll Mode

w_n(rad/sec)--------------------0.850 to 1.250

ξ------------------------------0.180 to 0.300

Rolling Mode

time constant (sec)-------------0.400 to 0.900

Spiral Mode

time constant (sec)-------------50.00 to 70.00

C_{x_u} ---------------------- -0.260 to -0.190

C_{z_u} ---------------------- -0.150 to 0.150

C_{m_u} ---------------------- 0.040 to 0.040

C_{x_α} ---------------------- 0.350 to 0.770

C_{z_α}	-7.700	to	-7.500
C_{m_α}	-1.780	to	-1.730
$C_{x_{\dot\alpha}}$	-0.040	to	0.100
$C_{z_{\dot\alpha}}$	-3.100	to	-2.850
$C_{m_{\dot\alpha}}$	-9.900	to	-9.200
C_{x_q}	-0.080	to	0.250
C_{z_q}	-7.700	to	-7.400
C_{m_q}	-25.000	to	-23.500
$C_{x_{\delta e}}$	-0.090	to	-0.060
$C_{z_{\delta e}}$	-0.564	to	-0.540
$C_{m_{\delta e}}$	-1.644 (constant)		
C_{y_β}	-0.749	to	-0.740
C_{l_β}	-0.050	to	-0.045
C_{n_β}	0.088	to	0.093
C_{y_p}	-0.134 (constant)		
C_{l_p}	-0.550	to	-0.535
C_{n_p}	-0.170	to	-0.110
C_{y_r}	0.388	to	0.396
C_{l_r}	0.396	to	0.420
C_{n_r}	-0.173	to	-0.155
$C_{l_{\delta a}}$	-0.075	to	-0.065
$C_{n_{\delta a}}$	0.00328	to	0.00344
$C_{y_{\delta r}}$	0.264 (constant)		
$C_{l_{\delta r}}$	0.0264 (constant)		
$C_{n_{\delta r}}$	-0.0920	to	-0.0910

V. Conclusions and Recommendations

The only dramatic differences between the two AC130E models occurred in cross-coupled responses which were at least one order of magnitude less than their respective primary perturbations. It is concluded that the existing model is reasonably accurate, based on available data; however, the qualitative comparison provided the following suggestions:

1. Significant cross-coupling occurs in the $\frac{\theta}{\delta a}$, $\frac{\theta}{\delta r}$, $\frac{\phi}{\delta e}$, and $\frac{\psi}{\delta e}$ responses, but these are unaffected by the gyroscopic coupling due to propellers. On this basis, the gyroscopic coupling is negligible; however, if other cross-coupling is to be considered, these effects should be included, as the phenomenon substantially increases the dutch roll in several of the smaller cross-coupled reactions.

2. The u-derivatives should be more rigidly defined and the contribution to C_{x_u} and C_{z_u} due to $\partial\alpha/\partial u$ in the body axis system, as well as the quantitative change in C_{m_u} due to axis transformation from stability to body axes, should be taken into account. These effects were not included in the existing model.

3. Flight test verification is needed to determine values for the rotary derivatives. None of the values used in either model (except C_{l_p}) can be analytically justified as all were obtained from empirical formulae.

Further study is necessary to provide information in the following areas:

1. Further computer simulation using different combinations of control deflections.

2. Possible changes in stability parameters to account for aeroelastic phenomena and the effects thereof on model response.

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Appendix A

Basic Data

(from Ref 1, 2, and 3)

	Wing	Horizontal Tail	Vertical Tail	Units
Area	1745.50	545.00	300.00	ft^2
Span	132.60	52.70	23.10	ft
MAC	13.71	11.18	14.82	ft
AR	10.09	5.02	1.78	
TR	0.52	0.37	0.30	
Dihedrel	1.50	0.00	N/A	degrees
Incidence	1.50	-1.75	N/A	degrees
Twist	-3.00	0.00	N/A	degrees

Fuselage

Max Length	97.74	ft
Frontal Area	180.00	ft^2
Horizontal Tail Length	47.33-C.G.(13.71)	ft
Vertical Tail Length	46.03-C.G.(13.71)	ft

Propeller

RPM		106.80	rad/sec
Diameter	A Model	16.00	ft
	E Model	13.50	ft
I	A Model	139.00	$slug/ft^2$
	E Model	175.40	$slug/ft^2$

$$T_c = 0.375-0.01A-(0.00338-0.0009A)\Delta V \qquad (a)$$

$$T_c = 0.275-0.008(A-10)-[0.00248-0.00007(A-10)]\Delta V \qquad (b)$$

where (a) is valid for $A \leq 10000$ ft, (b) is valid for $10000 \text{ ft} < A \leq 20000$ ft, and V = TAS-120 (knots).

$$C_T = \frac{8d^2}{S} T_c \rightarrow \begin{array}{ll} C_T = 1.032\ T_c & \text{A Model} \\ C_T = 0.836\ T_c & \text{E Model} \end{array}$$

$$C_L = C_{L_o} + \alpha C_{L_\alpha}, \quad \begin{array}{l} C_{L_o} = 0.25 + 0.12T_c + 0.90F \\ C_{L_\alpha} = 6.30 + 0.435T_c + 2.00F,\ M \leq 0.225 \\ C_{L_\alpha} = C_{L_{\alpha(0.225)}}[1 + 0.33\ (M-0.225)],\ M > 0.225 \end{array}$$

$$C_D = [C_{D_o} + K(C_L^2-0.20)](1+0.18FC_T), \quad \begin{array}{l} C_{D_o} = 0.030 + 0.048F \\ K = 0.044 - 0.030F \end{array}$$

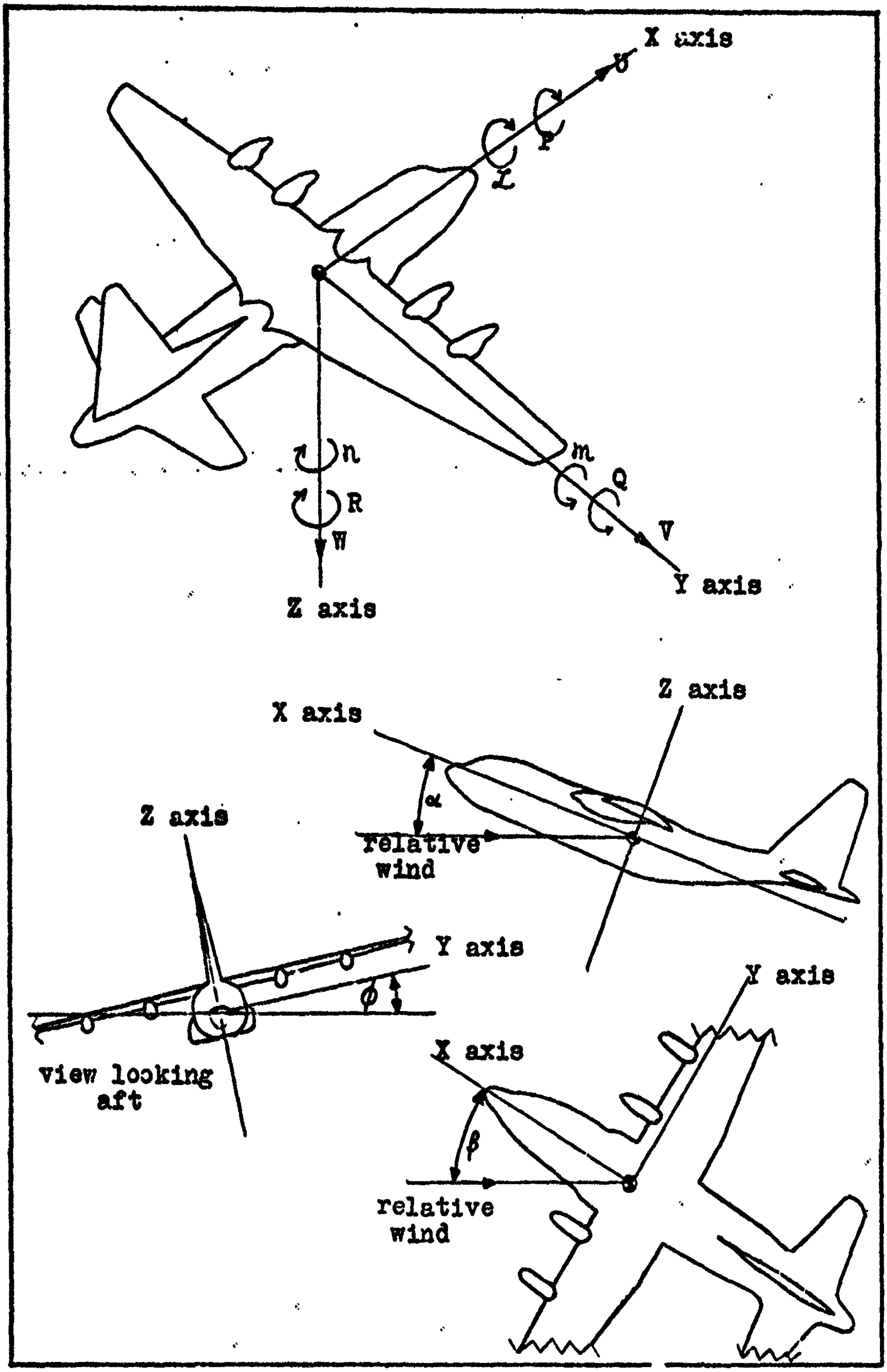


Figure 1. Body Axis System

Appendix B

C130E Model Response to Control Deflection

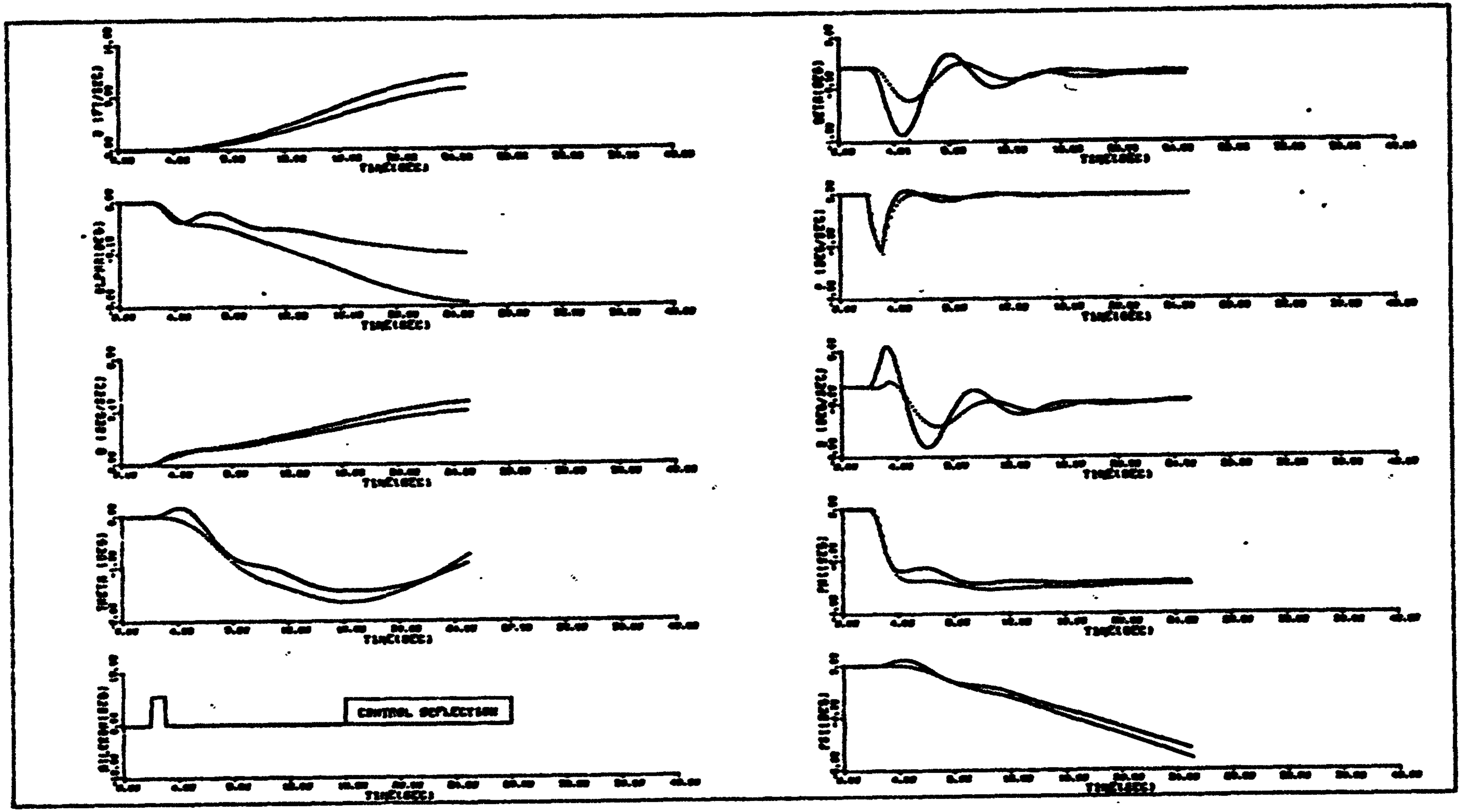


FIG. 2 : AIRCRAFT RESPONSE TO CONTROL SURFACE DEFLECTION

LEGEND: PRESENT MODEL . . . .
PROPOSED MODEL ————————

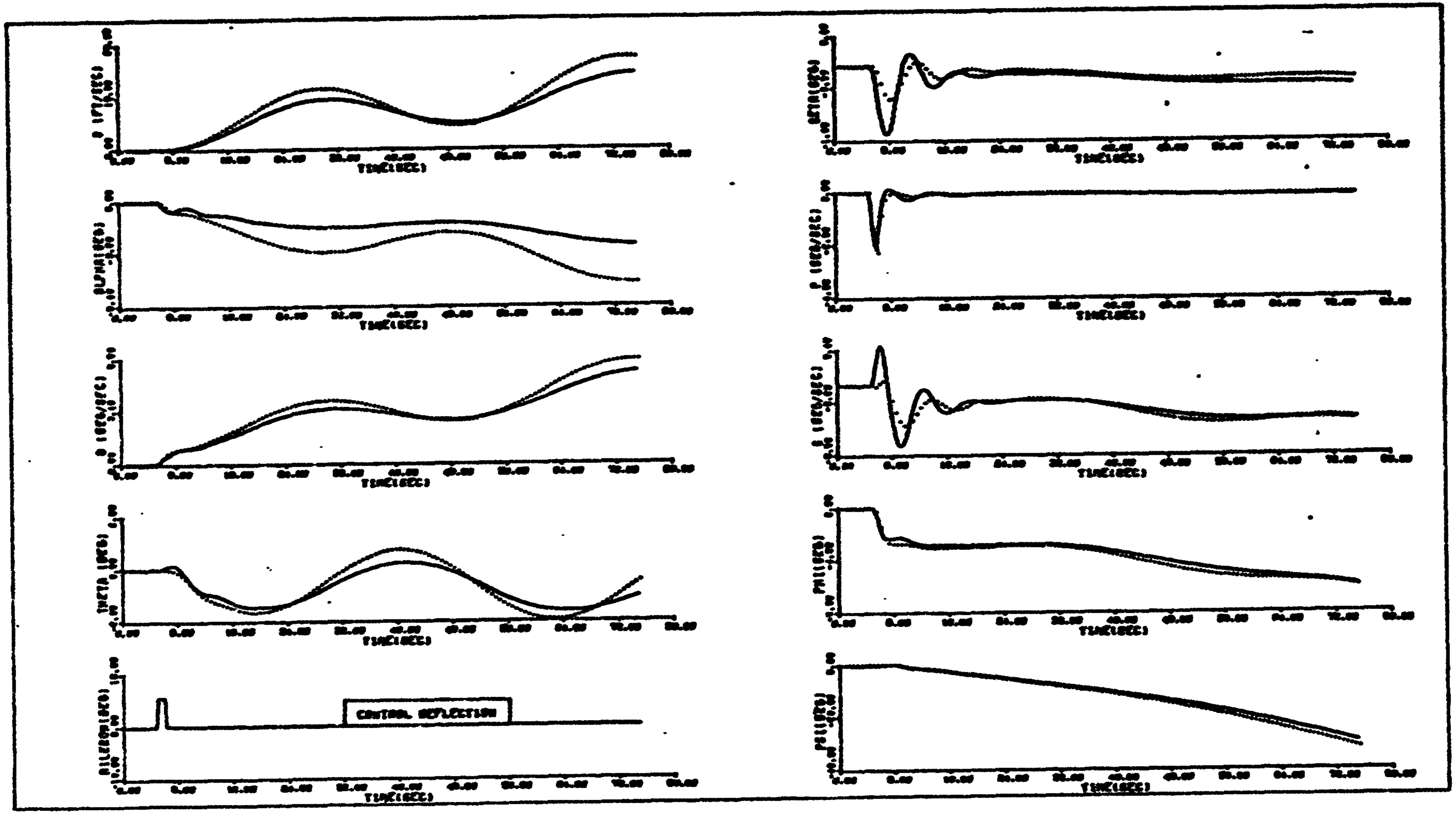


FIG. 3. AIRCRAFT RESPONSE TO CONTROL SURFACE DEFLECTION

LEGEND: PRESENT MODEL ....
PROPOSED MODEL ————

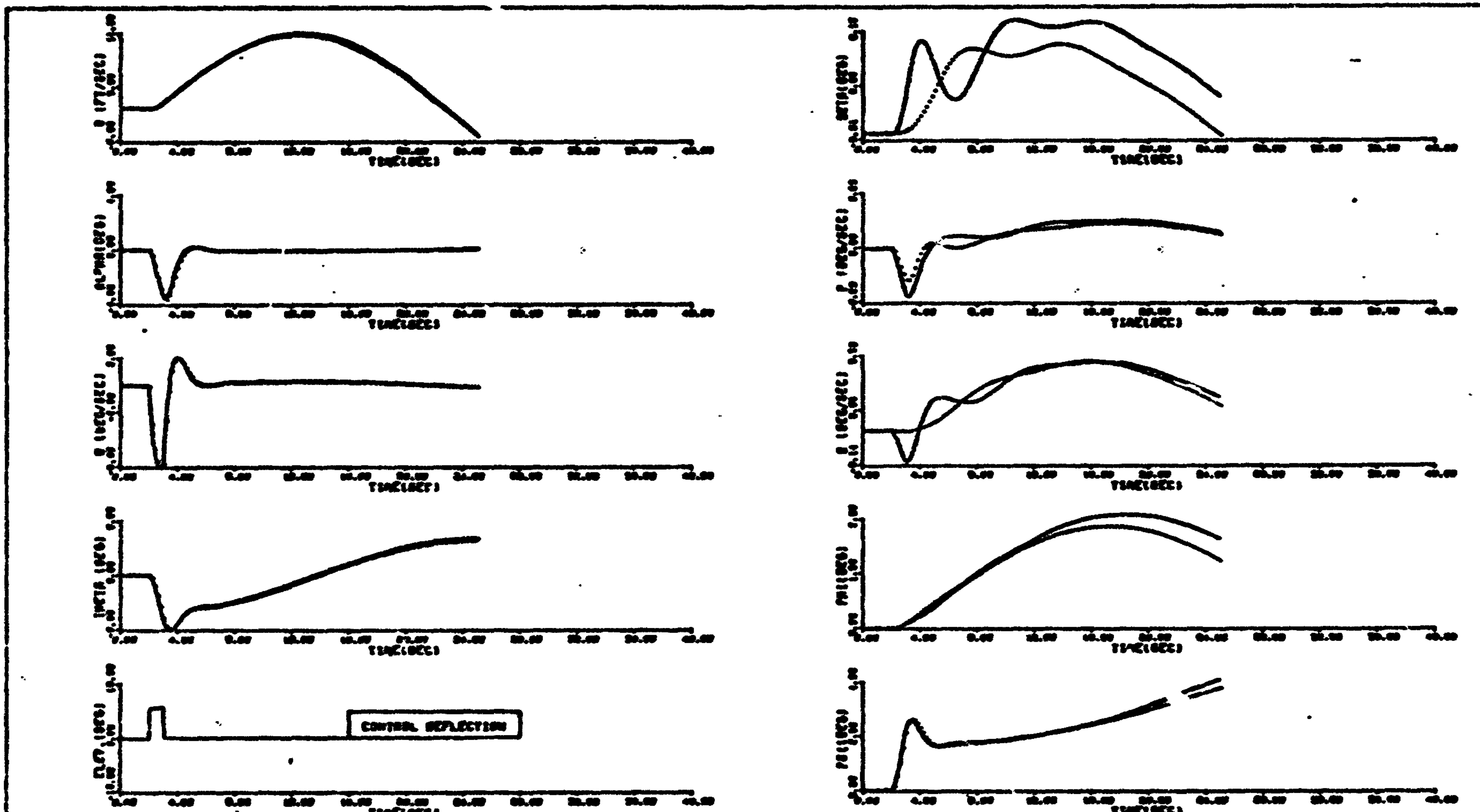


FIG. 4 : AIRCRAFT RESPONSE TO CONTROL SURFACE DEFLECTION

LEGEND: PRESENT MODEL ....
PROPOSED MODEL ————

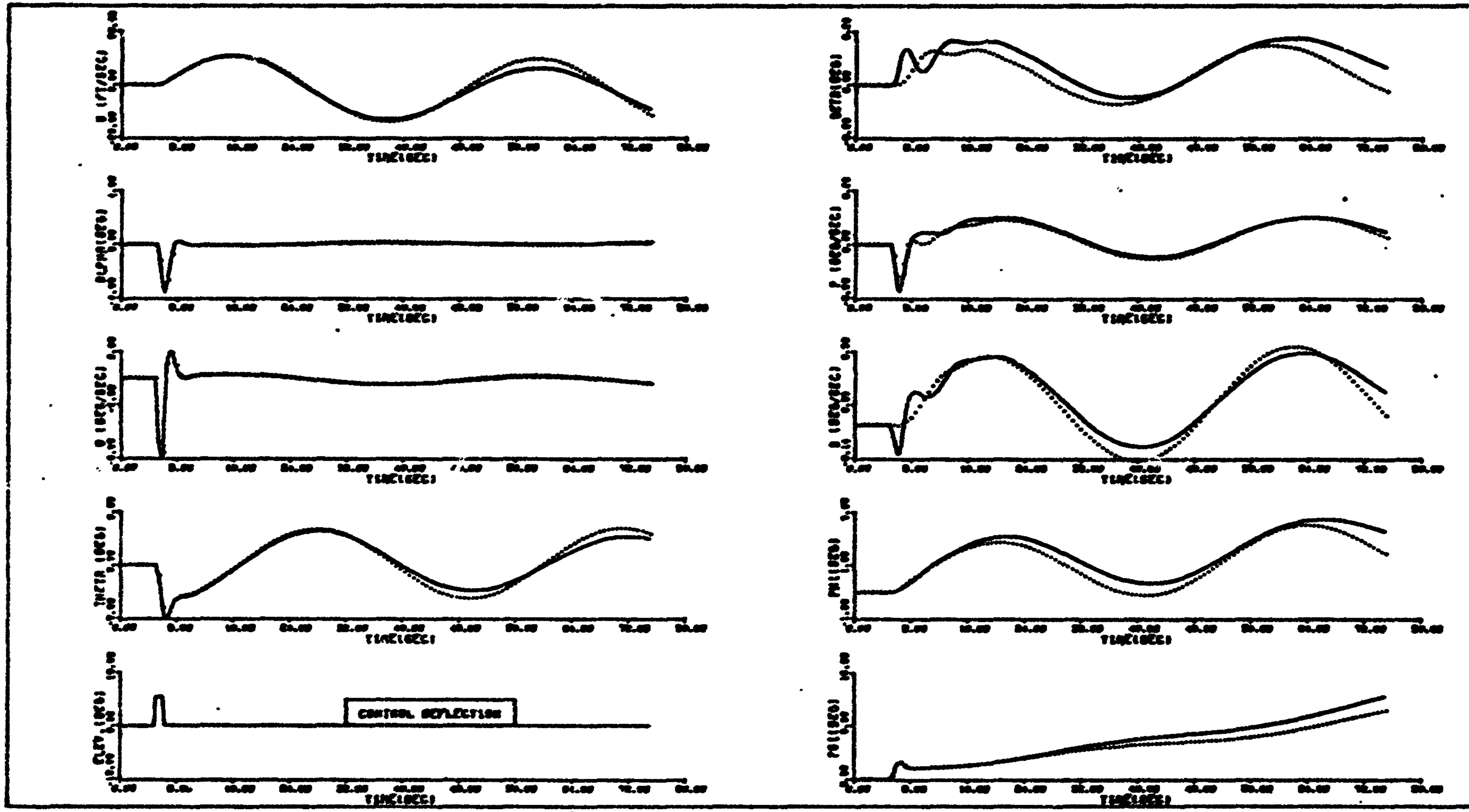


FIG. 5 . AIRCRAFT RESPONSE TO CONTROL SURFACE DEFLECTION

LEGEND: PRESENT MODEL . . . .
PROPOSED MODEL ————

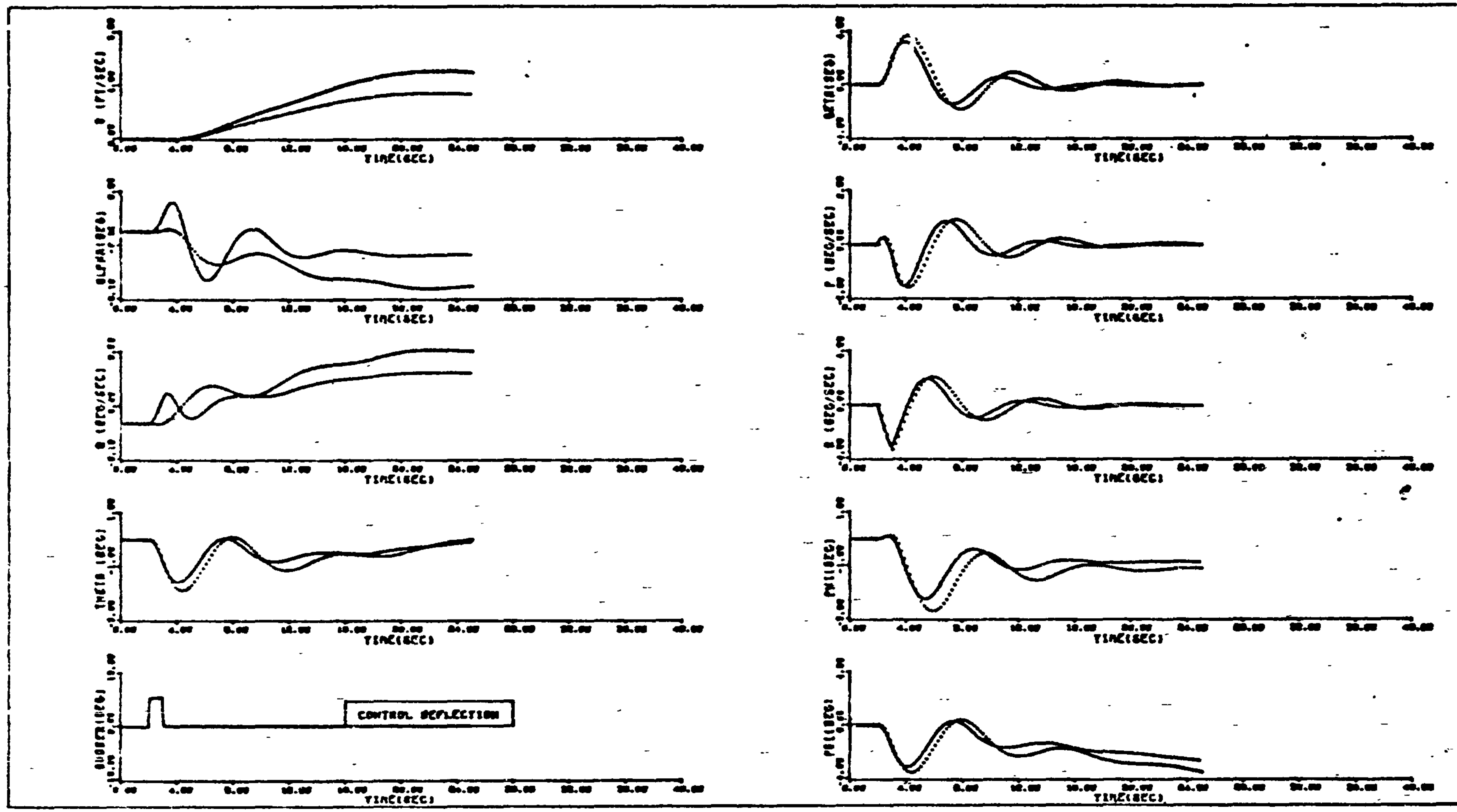


FIG. 6 , AIRCRAFT RESPONSE TO CONTROL SURFACE DEFLECTION

LEGEND: PRESENT MODEL ....
PROPOSED MODEL ——————

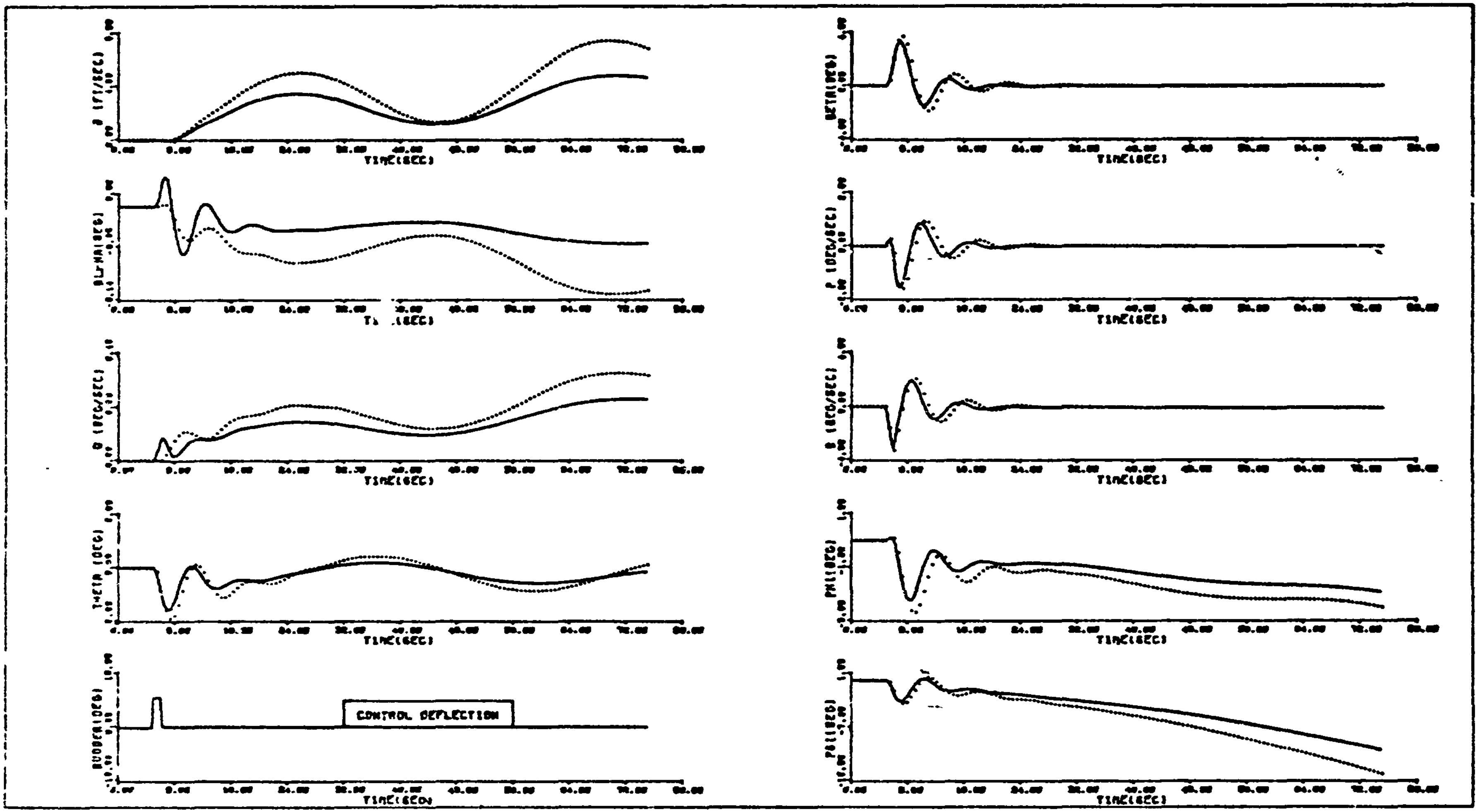
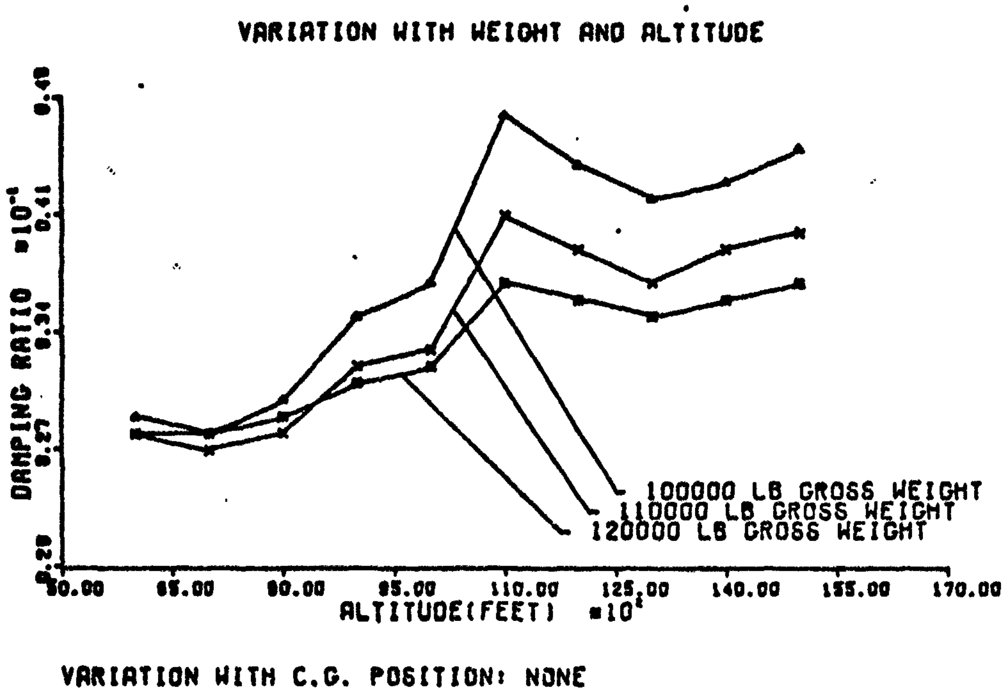


FIG. 7 . AIRCRAFT RESPONSE TO CONTROL SURFACE DEFLECTION

LEGEND: PRESENT MODEL . . . .
PROPOSED MODEL ————————

Appendix C

Stability Parameters for the AC130A Aircraft



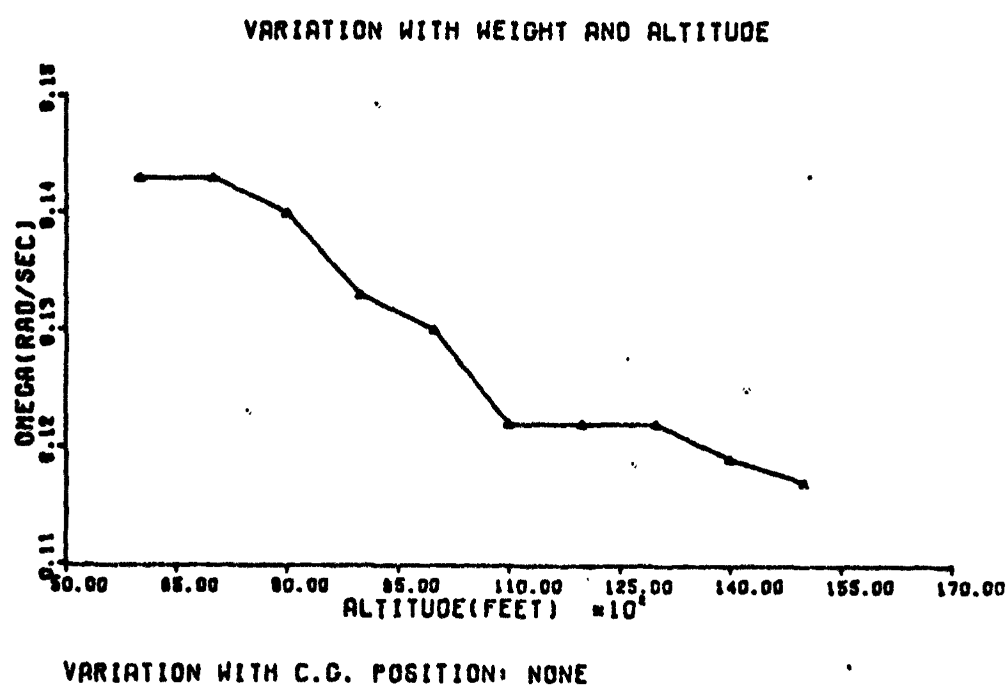


Figure 8. Phugoid Parameters

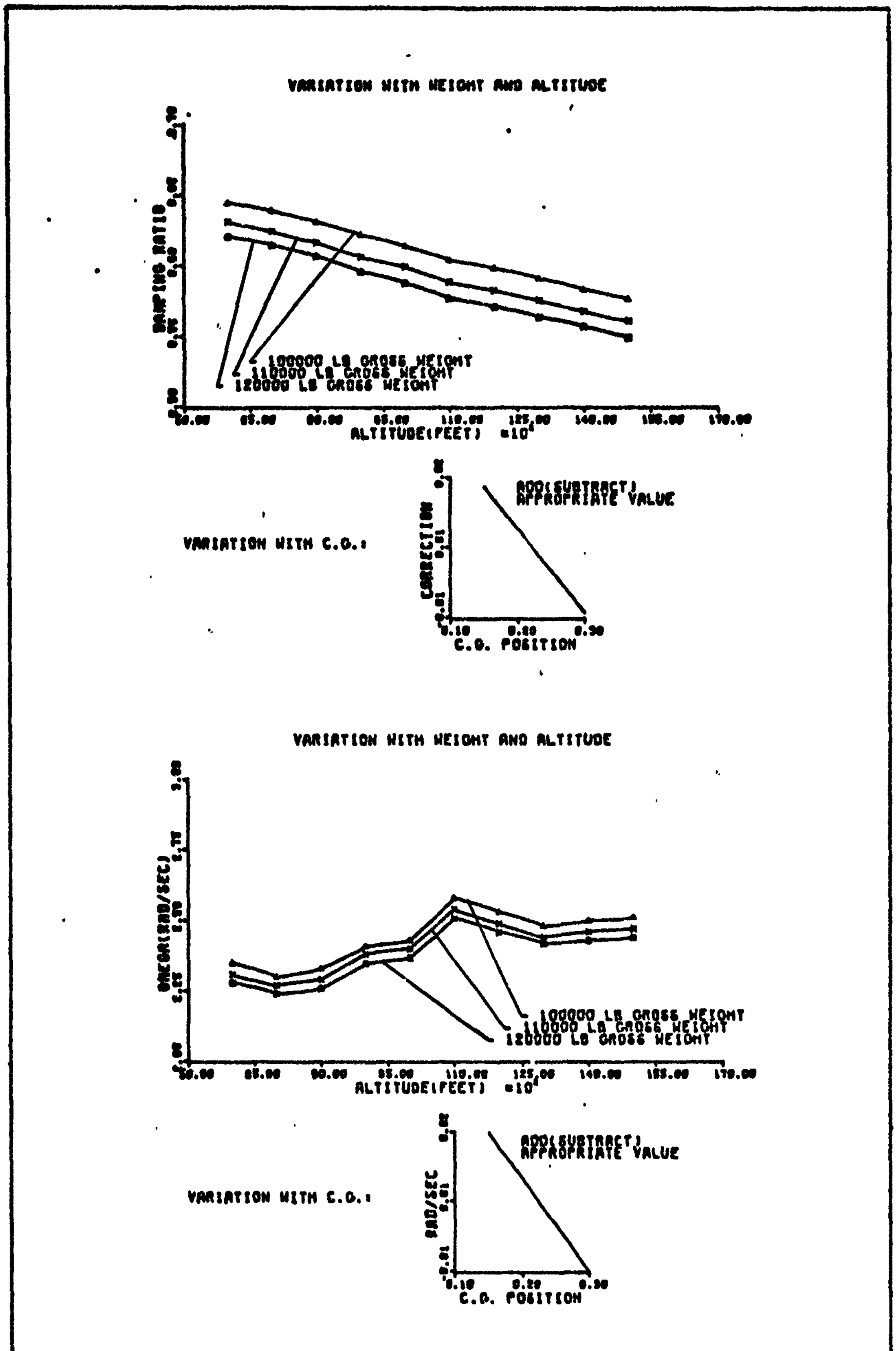


Figure 9. Short Period Parameters

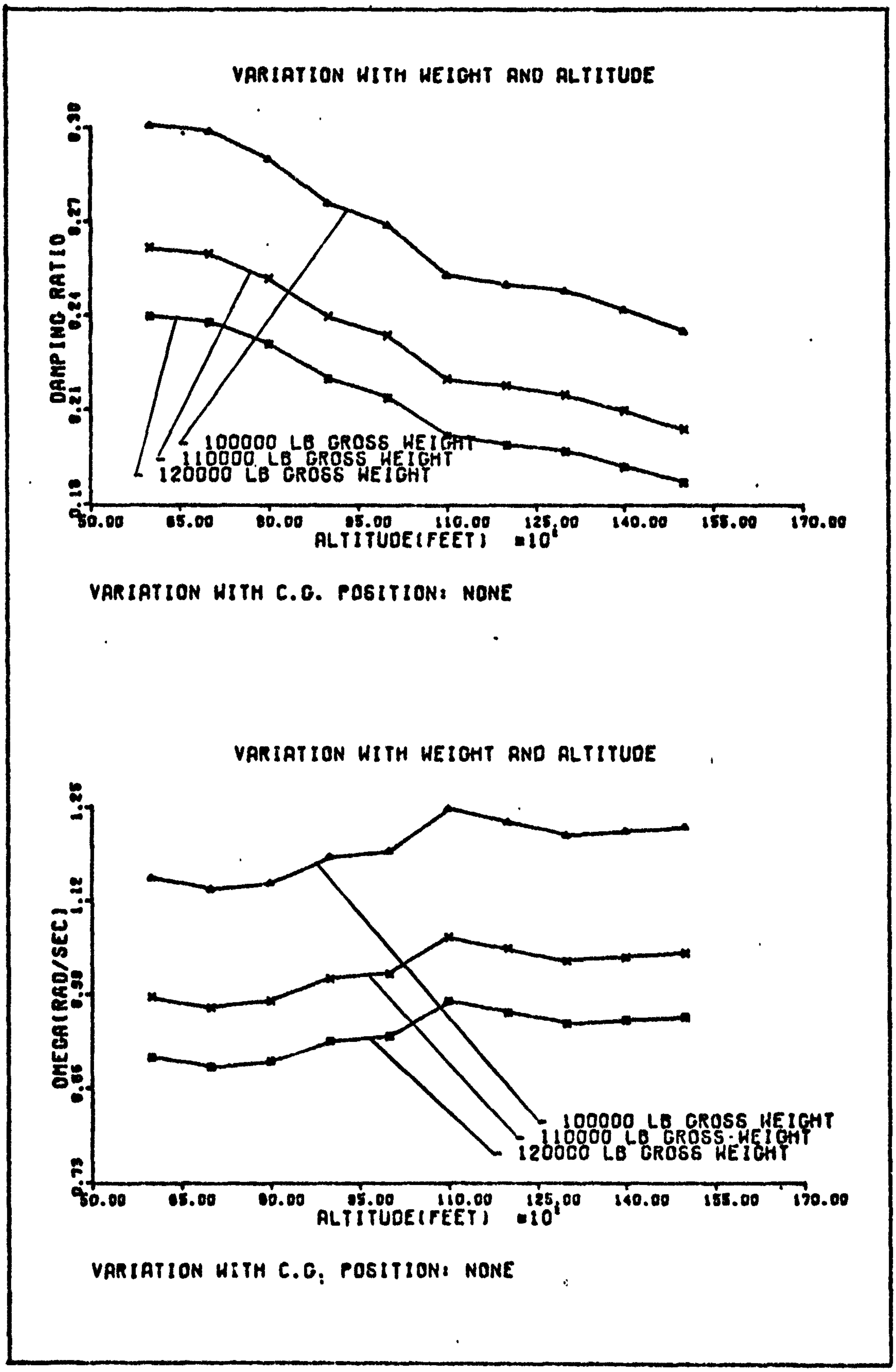
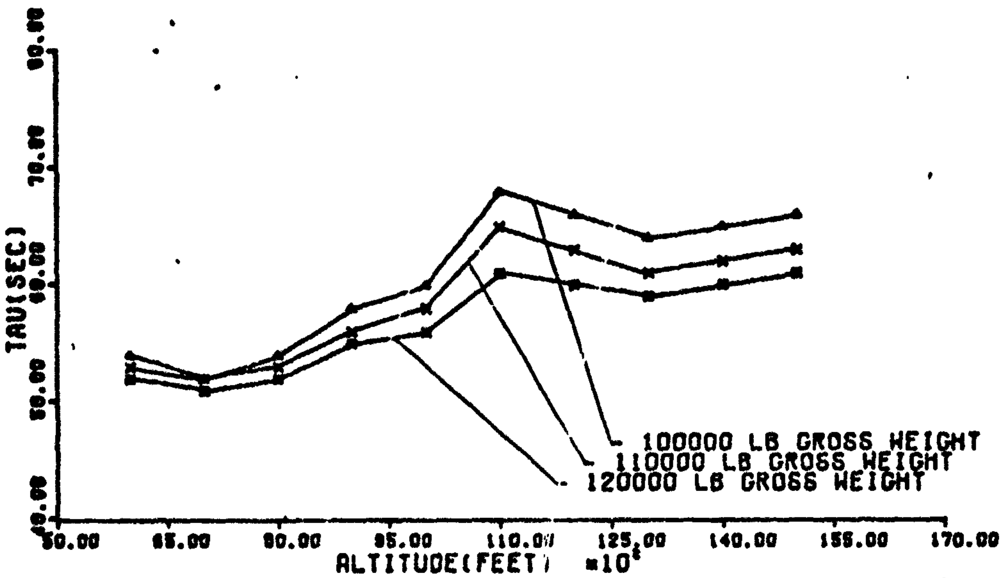


Figure 10. Dutch Roll Parameters

Spiral Mode

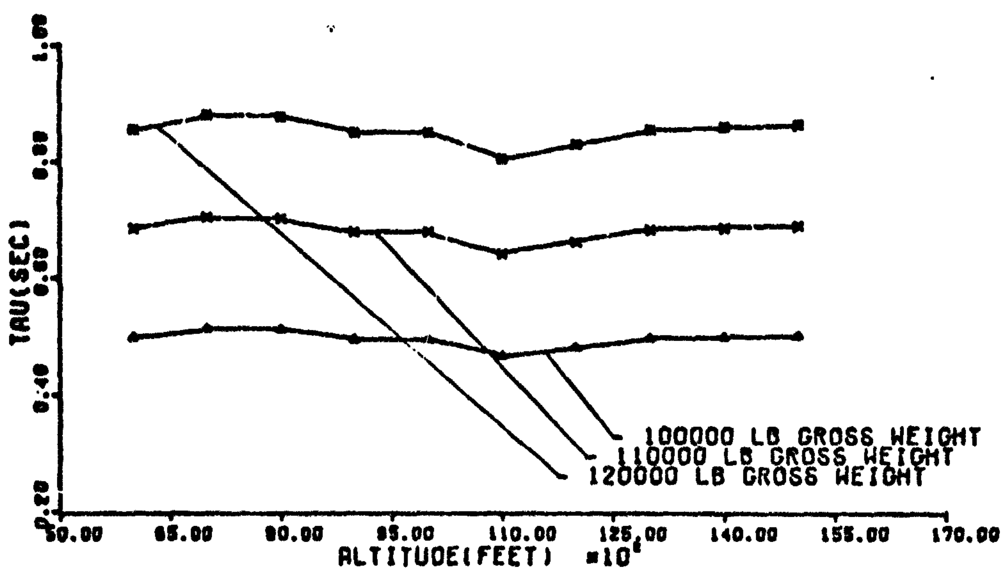
VARIATION WITH WEIGHT AND ALTITUDE



VARIATION WITH C.G. POSITION: NONE

Rolling Mode

VARIATION WITH WEIGHT AND ALTITUDE



VARIATION WITH C.G. POSITION: NONE

Figure 11. Spiral and Rolling Modes Parameters

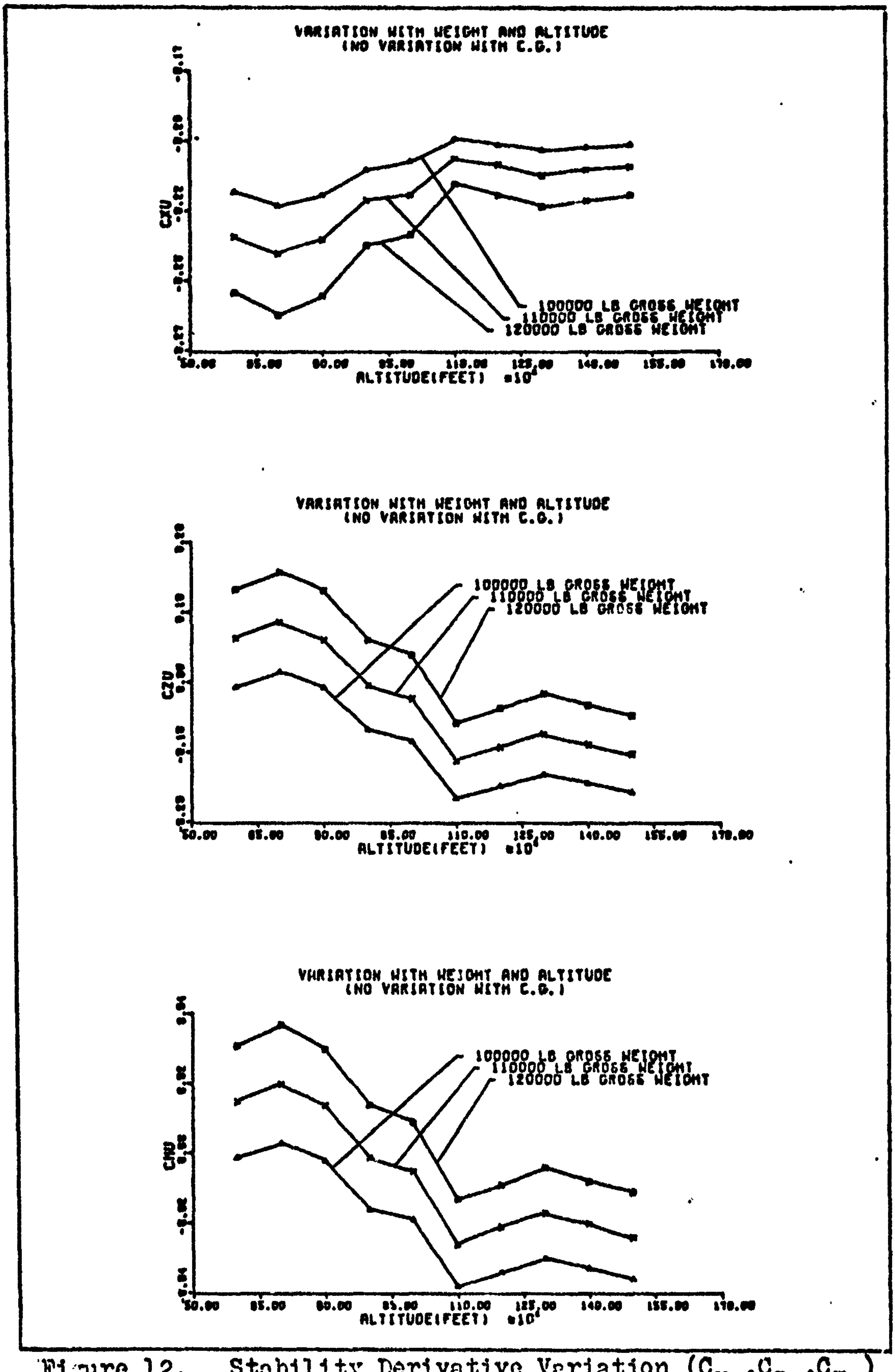


Figure 12. Stability Derivative Variation (C_{x_u}, C_{z_u}, C_{m_u})

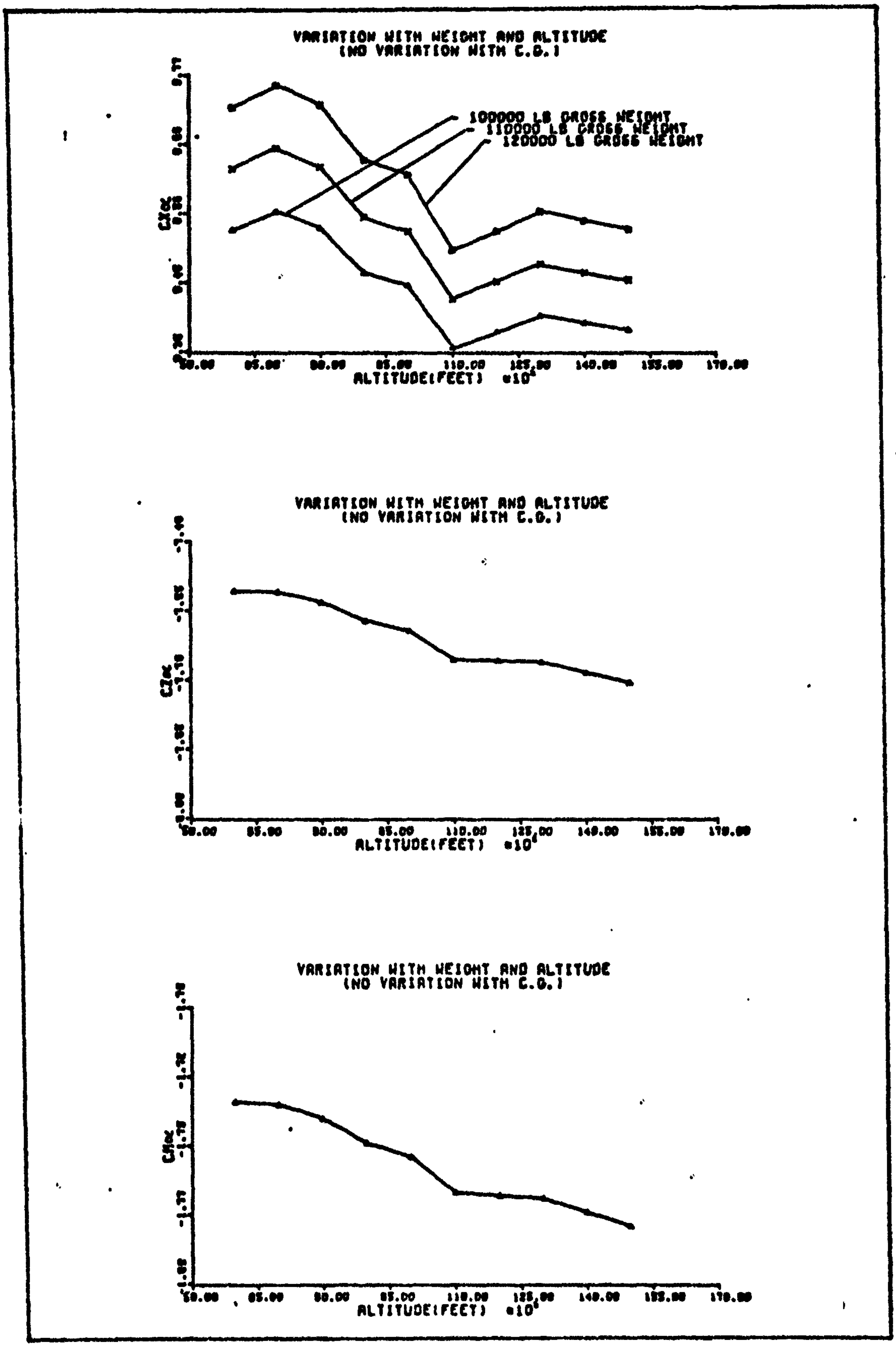


Figure 13. Stability Derivative Variation (C_{x_α}, C_{z_α}, C_{m_α})

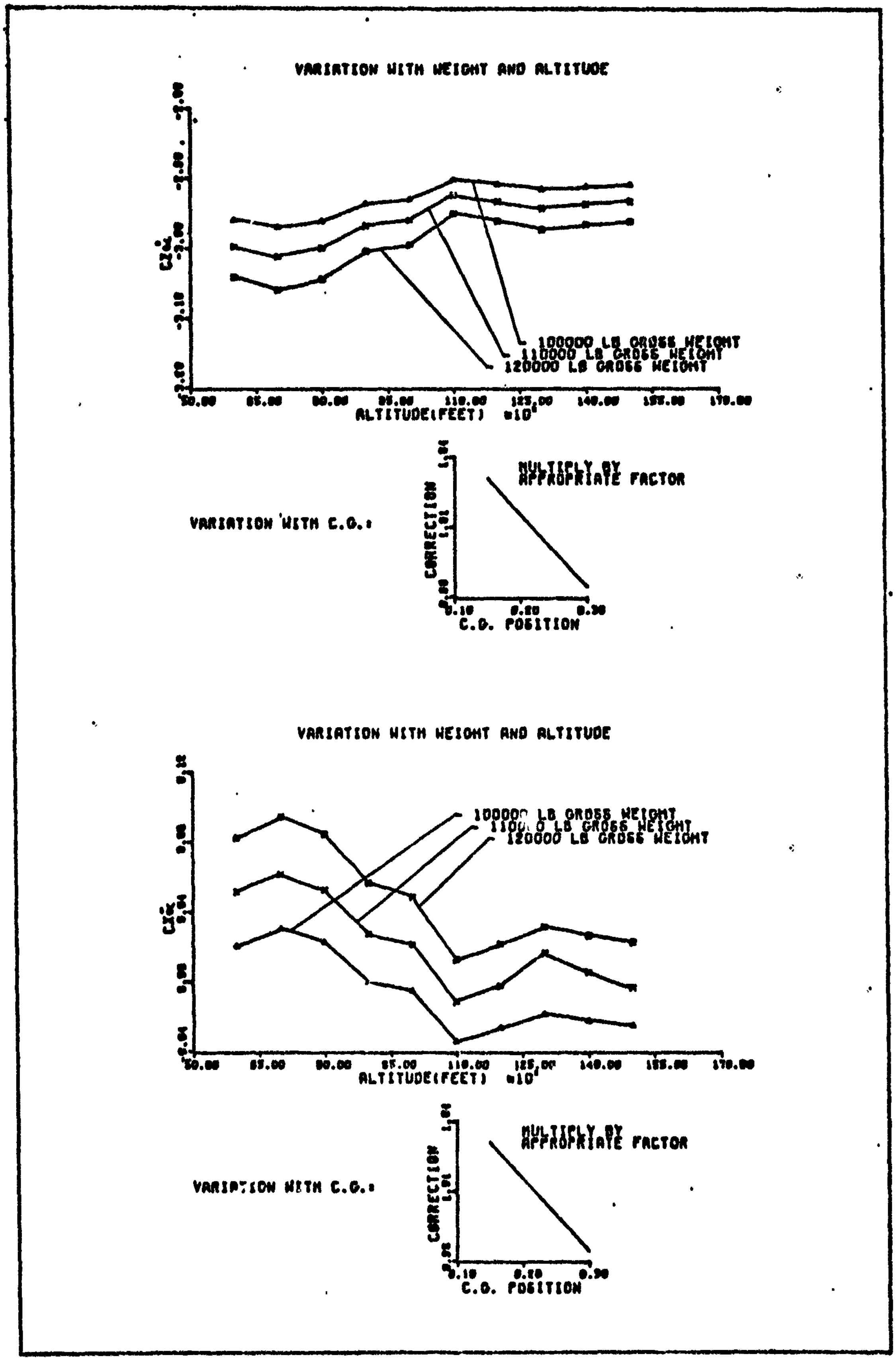


Figure 14. Stability Derivative Variation ($C_{x_{\dot{\alpha}}}, C_{z_{\dot{\alpha}}}$)

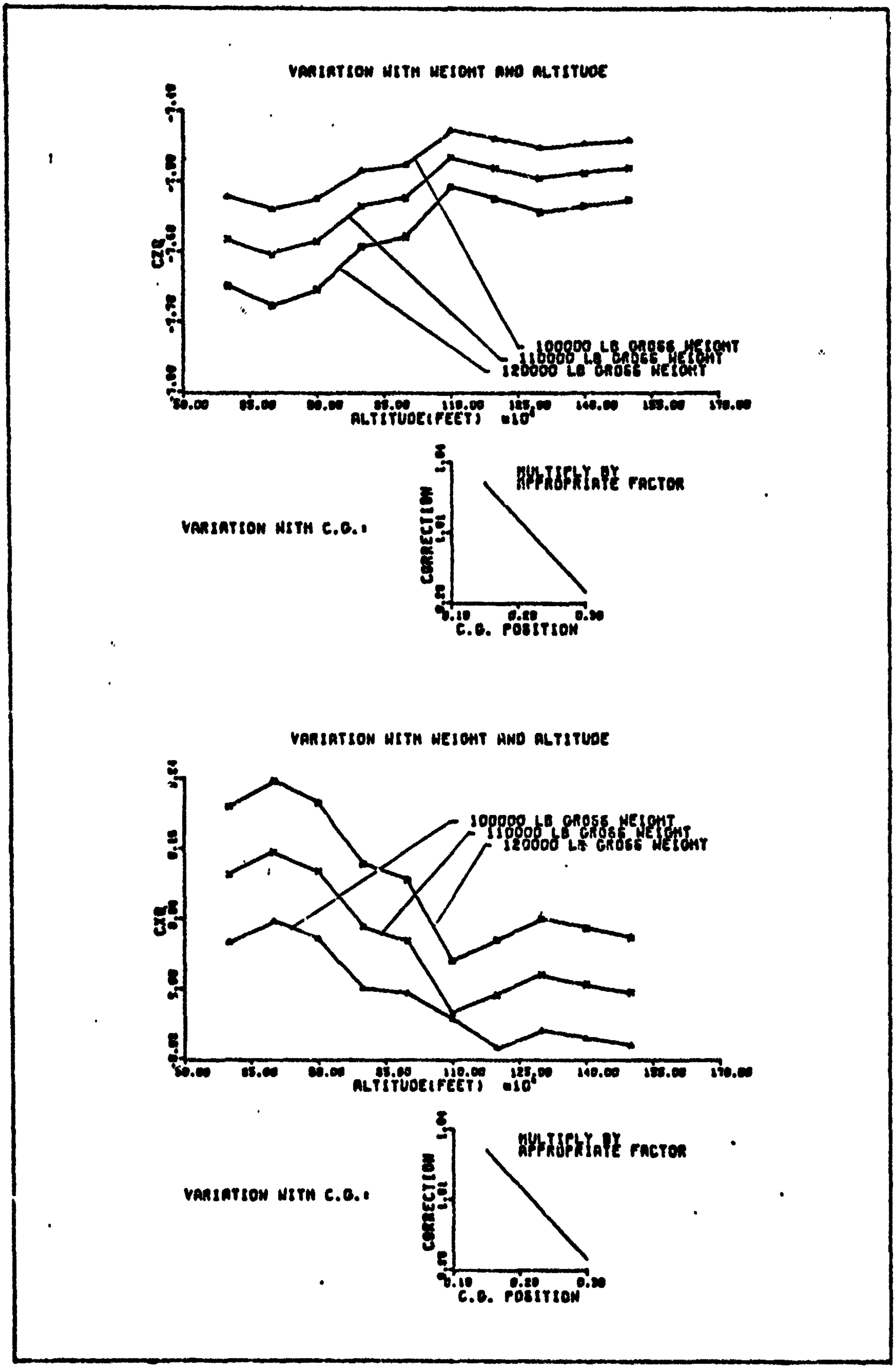


Figure 15. Stability Derivative Variation (C_{x_q},C_{z_q})

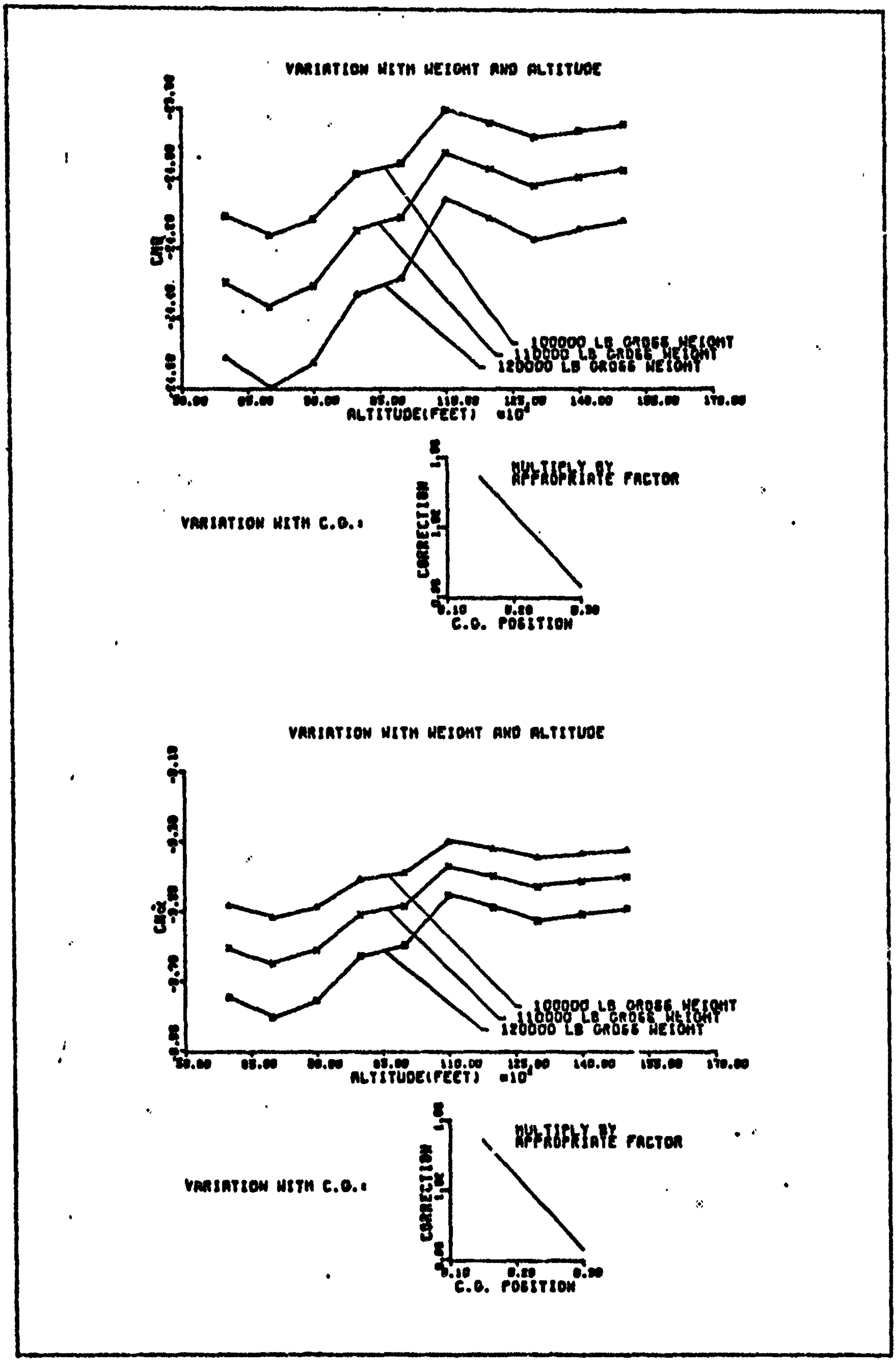


Figure 16. Stability Derivative Variation (C_{m_q}, $C_{m_{\dot{\alpha}}}$)

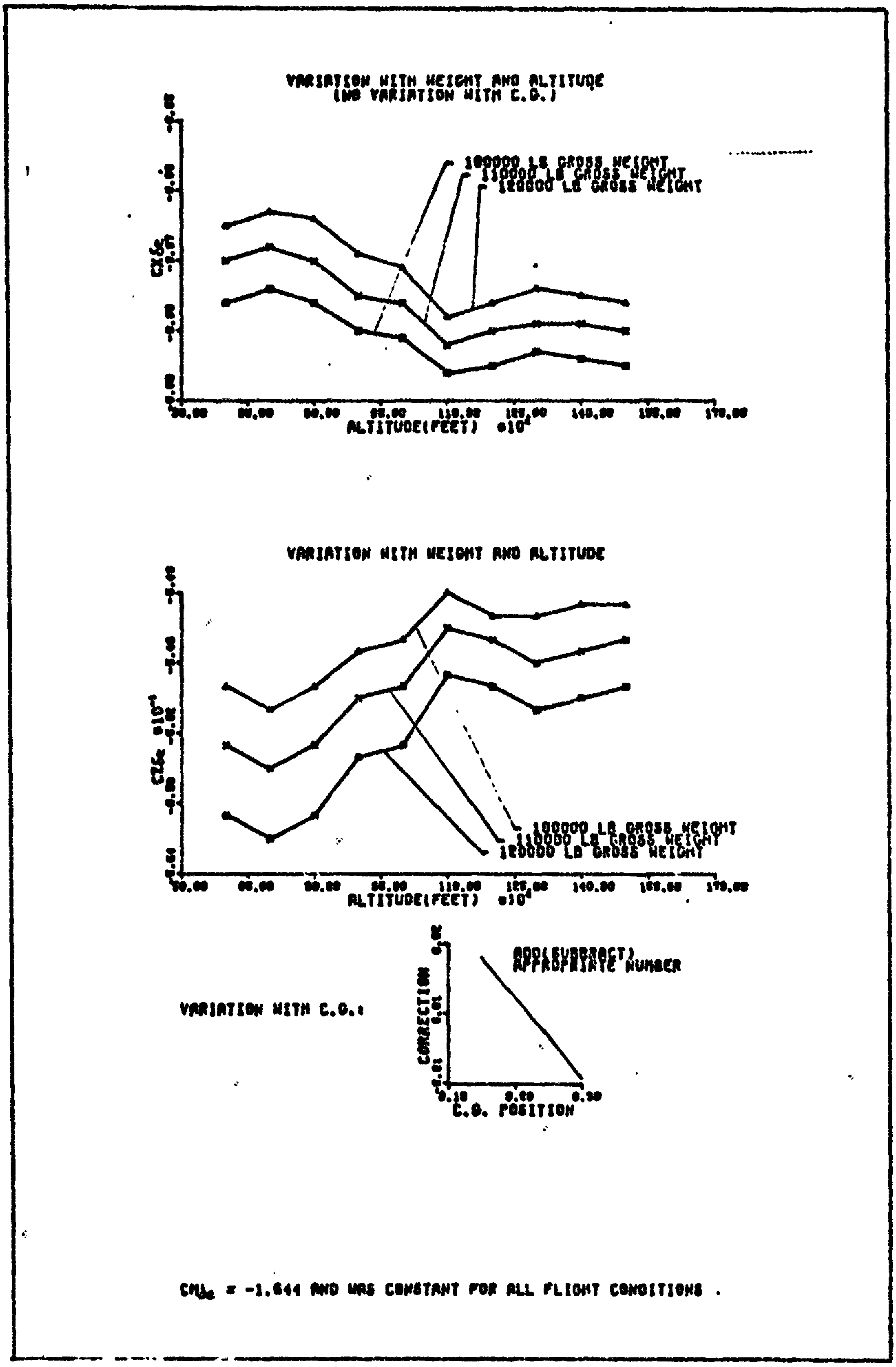


Figure 17. Stability Derivative Variation ($C_{x_{\delta_e}}, C_{z_{\delta_e}}, C_{m_{\delta_e}}$)

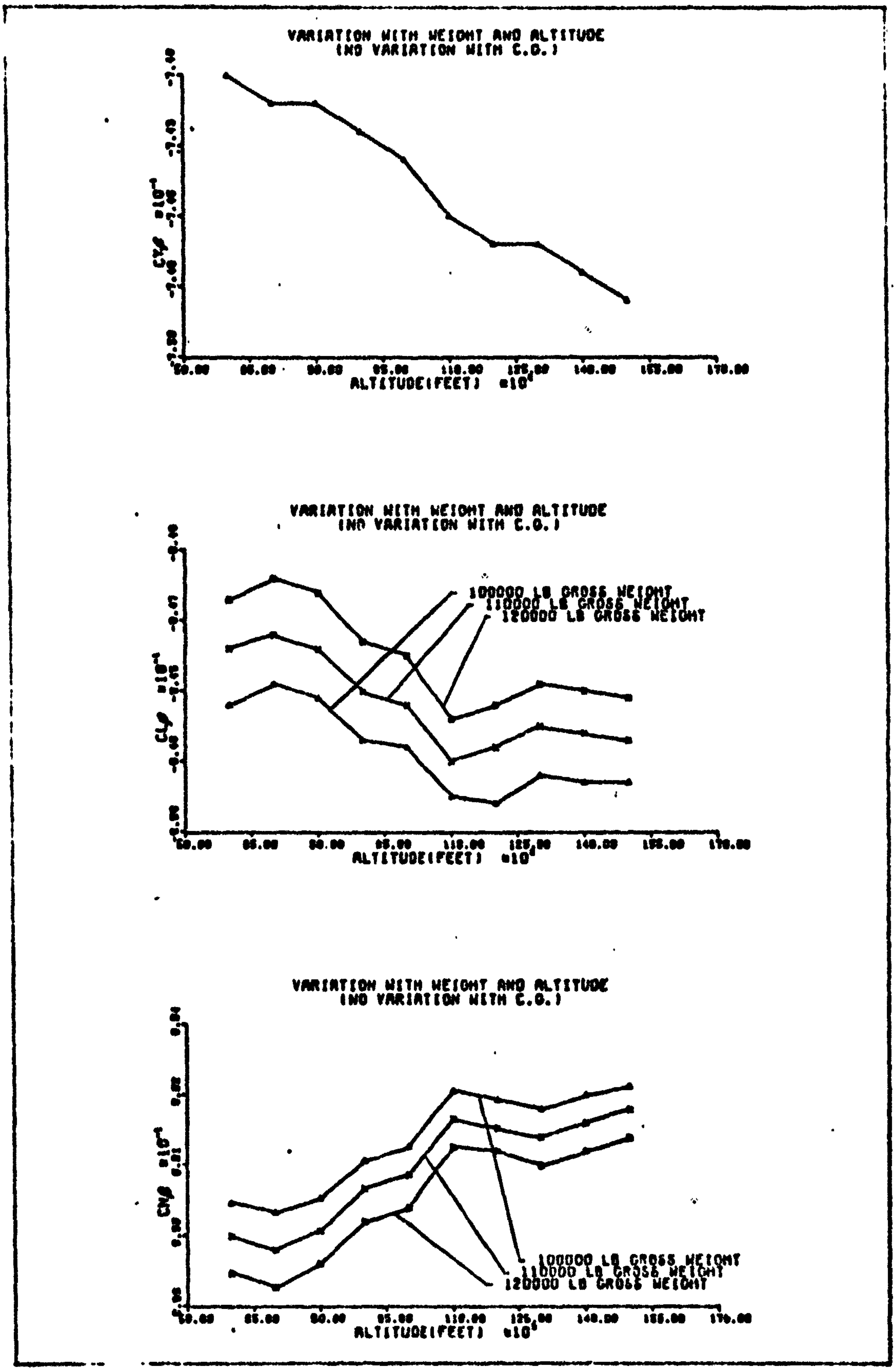


Figure 13. Stability Derivative Variation ($C_{y_\beta}, C_{l_\beta}, C_{n_\beta}$)

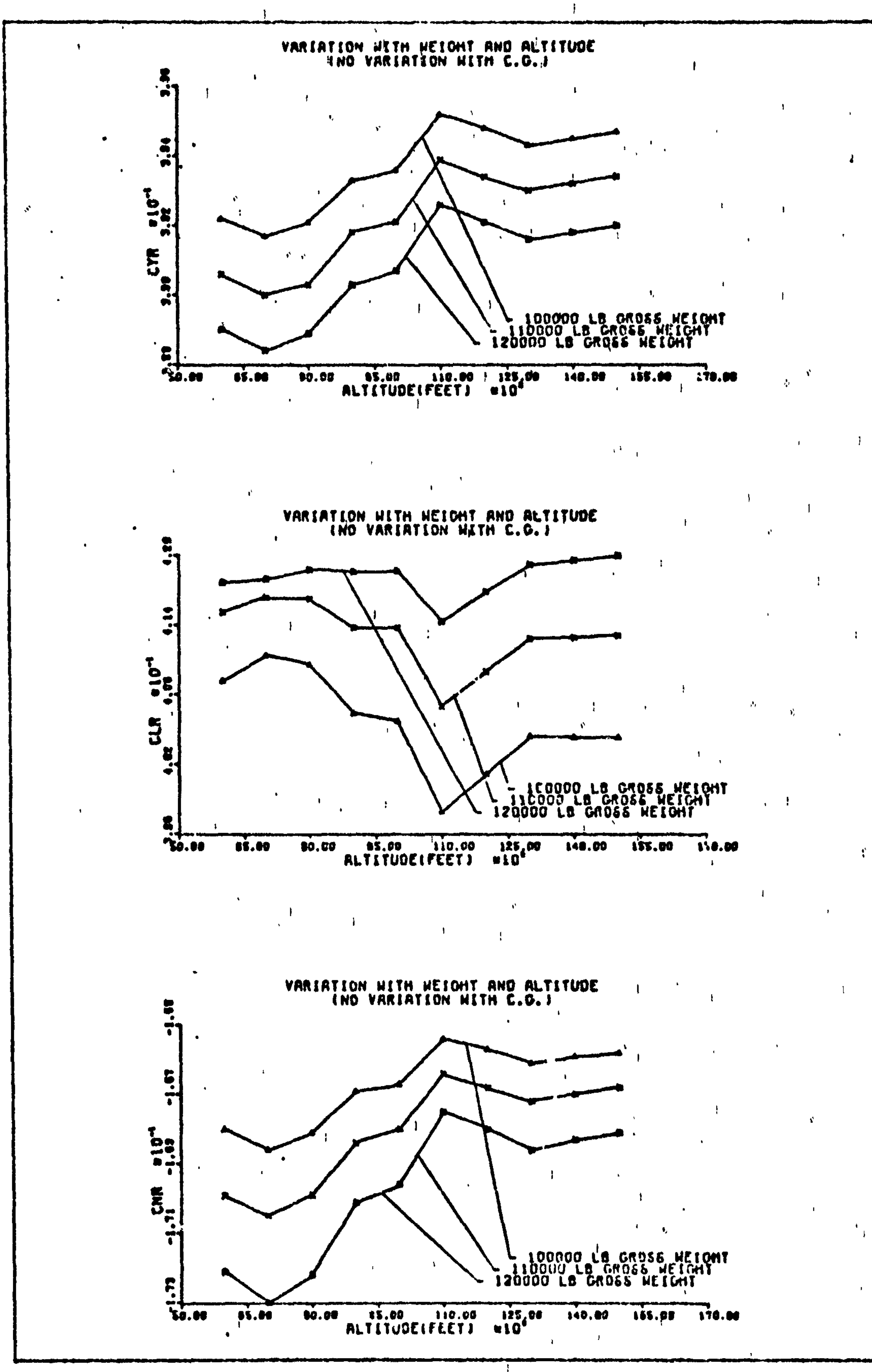


Figure 19. Stability Derivative Variation ($C_{y_r}, C_{l_r}, C_{n_r}$)

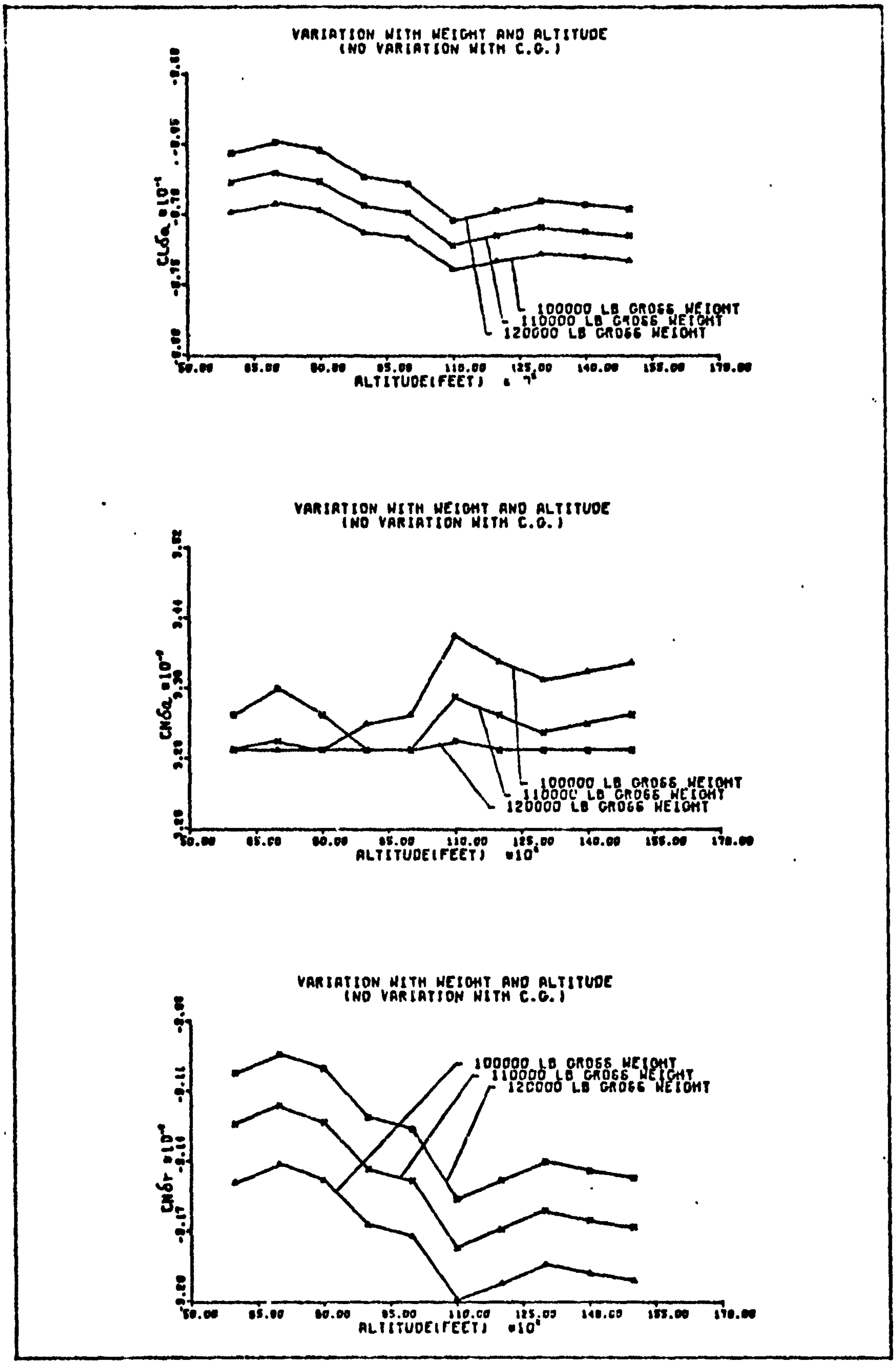
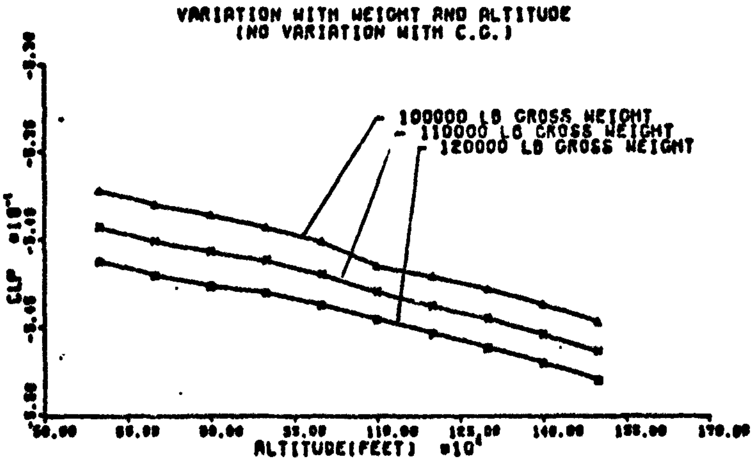
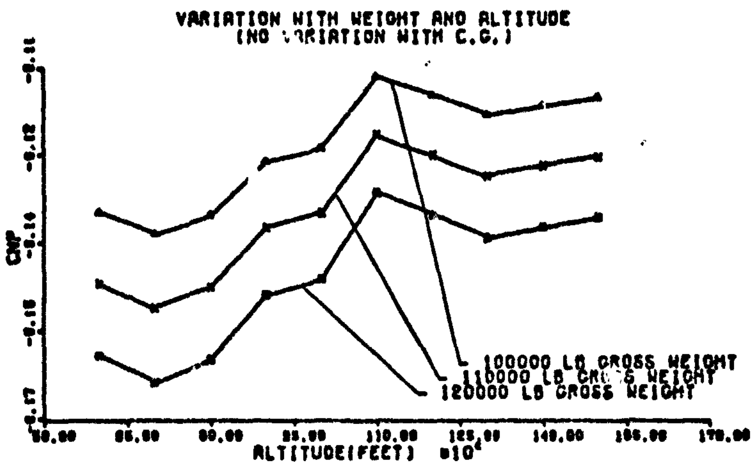


Figure 20. Stability Derivative Variation ($C_{l_{\delta_a}}, C_{n_{\delta_a}}, C_{n_{\delta_r}}$)





CYP = -0.134 AND WAS CONSTANT FOR ALL FLIGHT CONDITIONS

CYδr = 0.264 AND WAS CONSTANT FOR ALL FLIGHT CONDITIONS

CLδr = 0.0264 AND WAS CONSTANT FOR ALL FLIGHT CONDITIONS

Figure 21. Stability Derivative Variation $\left(\begin{matrix} C_{l_p}, C_{n_p}, C_{y_p} \\ C_{y_{\delta_r}}, C_{l_{\delta_r}} \end{matrix}\right)$

Appendix D

Computer Programs

```
PROGRAM COMPUTES INITIAL CONDITIONS,MODE PARAMETERS
AND PERTURBED RESPONSE:PRINT STATEMENTS NOT INCLUDED
ESTIMATE 65K MEMORY AND 4JL SECONDS
THIS PROGRAM IS SET UP FOR THE = MODEL
IF A MODEL IS USED, CHANGE FOLLOWING TERMS...
TC=T/(K1*1.332),  CT=1.672*TC,  I=139.C
      DIMENSION X1(255),X2(255),Y3(255),X4(255),X5(255),
     *Y6(255),X7(255),X8(255),X9(255),X10(255),
     *X11(255),X12(255)
      DIMENSION T1(255),Y(12)
      COMMON AA(9,12),Y(255,12)
      DIMENSION WN(9),ZETA(9),PD(9),THALF(9),APFAL(2)
      REAL K,K1,K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13,K14,
     *K15,K16,K17,K18,K20,K21,K22,K23,K24,K25,K26,K27,K28,K29
      REAL I,IXX,IYY,IZZ,IXZ,M,M1,LT,LF
INPUT PARAMETERS WHICH ARE CONSTANT
      READ 2L3,G,S,B,CHAR,PI,RPM,AR
INPUT PARAMETERS WHICH VARY WITH FLIGHT CONDITION
      READ 1.7,U,ALTO,PHI,FLAPS,W,TEMP
WHERE U IS TAS IN KNOTS,ALTO IS PRESS ALTITUDE
IN FEET,PHI IS BANK ANGLE IN DEGREES,W IS GROSS
WEIGHT IN LB,AND TEMP IS FREE STREAM AIR TEMP
      READ 105,IXX,IYY,IZZ,IXZ,I,CG
COMPUTE PARAMETERS NEEDED TO DETERMINE AERODYNAMIC DATA
      RPM=1323.0*PI/30.0
      PHI=PHI/57.29578
      M=W/G
      RHO=.0.23769*(1.0-.0000068825*ALTO)**4.255
      F=FLAPS/100.0
      DELV=U-120.L0
      U=U*6080.0/3600.0
      VSOUND=49.1*SQRT(TEMP)
      M1=U/VSOUND
      ALTO=ALTO/1000.0
      CDL=1.1*(.028+.052*F)
      K=.062-.02L*F
      CLMINDG=.25+.7*F
COMPUTE CONSTANTS
      K1=RHO*S*U**2/2.0
      K2=K1/M
      K3=RHO*S*U/(2.0*M)
      K4=K1*B
INITIAL VALUES OF A AND T TO BEGIN ITERATION
      IF (PHI.EQ.0.0) GO TO 46
      R1=U**2/(G*TAN(PHI))
      PSIDOT=U/R1
      GO TO 47
   46 PSIDOT=0.0
   47 CONTINUE
      IF (ALTO.GT.10.L) GO TO 21
      TC=.375-.01*ALTO-(.00338-.00039*ALTO)*DELV
      GO TO 22
   21 TC=.275-.008*(ALTO-10.0)
     *-(.00248-.000066*(ALTO-10.))*DELV
```

```
   22 CONTINUE
      CT=0.836*TC
      T=CT*K1
      CLL=W/(K1*COS(PHI))
      CLL0=.25+.12*TC+.90*F
      IF (M1.LT.0.225) GO TO 3
      CLLA=(6.3+.435*TC+2.0*F)*(1.0+.333(M1-.225))
      GO TO 4
    3 CLLA=6.3+.435*TC+2.0*F
    4 A=(CLL-CLL0)/CLLA
C ITERATION ON ALPHA AND THRUST USING CRAMERS RULE
      DO 10 J=1,5
      IF (J.LT.2) GO TO 14
      CLL0=.25+.12*TC+.90*F
      IF (M1.LT.0.225) GO TO 15
      CLLA=(6.3+.435*TC+2.0*F)*(1.0+.333(M1-.225))
      GO TO 14
   15 CLLA=6.3+.435*TC+2.0*F
   14 CLL=CLL0+CLLA*A
      CD=CD0+K*(CLL-CLMINDG)**2+.005
      CT=T/K1
      CD=CD*(1.+.18*F*CT)
      CDA=2.*K*CLLA*(CLL-CLMINDG)
      CX=CLL*SIN(A)-CD*COS(A)
      CXA=(CLL-CDA)*COS(A)+(CD+CLLA)*SIN(A)
      CZ=-CLL*COS(A)-CD*SIN(A)
      CZA=(CLL-CDA)*SIN(A)-(CD+CLLA)*COS(A)
      TH=ATAN(TAN(A)*COS(PHI))
      P=-PSIDOT*SIN(TH)
      Q=PSIDOT*COS(TH)*SIN(PHI)
      R=PSIDOT*COS(TH)*COS(PHI)
      UDOT=K2*CX+T/M-G*SIN(TH)-U*Q*A
      ADOT=K2*(CZ-A*CX)-(A*T)/(M*U)+Q*(1.+A**2)
     *+(G/U)*(COS(TH)*COS(PHI)+A*SIN(TH))
      DTHDA=COS(PHI)/(1.0-(SIN(PHI)*SIN(A))**2)
      DPDA=-PSIDOT*COS(TH)*DTHDA
      DQDA=-PSIDOT*SIN(PHI)*SIN(TH)*DTHDA
      DRDA=-PSIDOT*COS(PHI)*SIN(TH)*DTHDA
      DUDDA=K2*CXA-G*COS(TH)*DTHDA-U*(Q+A*DQDA)
      DUDDT=1.0/M
      DADDA=K2*(CZA-CX-A*CXA)-T/(M*U)+(G/U)*(SIN(TH)
     *+A*COS(TH)*DTHDA-SIN(TH)*COS(PHI)*DTHDA)+2.0*A*Q
     *+DQDA*A**2
      DADDT=-A/(M*U)
      DET=DUDDA*DADDT-DUDDT*DADDA
      DELA=(DUDDT*ADOT-DADDT*UDOT)/DET
      DELT=(DADDA*UDOT-DUDDA*ADOT)/DET
      A=A+DELA
      T=T+DELT
      TC=T/(K1*.836)
   10 CONTINUE
C TRANSFORM INERTIA TERMS TO BODY AXIS
      ALPHA=A*57.29578
      IXZ=IXZ*(1.-ALPHA/4.35)
```

```
      IXXS=IXX
      IZZS=IZZ
      IXZS=IXZ
      IXX=IZZS*SIN(A)**2+IXXS*COS(A)**2+2.*IXZS*SIN(A)*COS(A)
      IZZ=IXXS*SIN(A)**2+IZZS*COS(A)**2-2.*IXZS*SIN(A)*COS(A)
      IXZ=IXZS*(COS(A)**2-SIN(A)**2)+(IZZS-IXXS)*SIN(A)*COS(A)
C COMPUTE CONSTANTS
      K5=IXZ/IZZ
      K6=IYY-IZZ+(IXZ**2/IZZ)
      K7=IXZ*(1.0+(IXX-IYY)/IZZ)
      K8=K5*I*RPM*4.0
      K9=K1*CBAR/IYY
      K10=(IZZ-IXX)/IYY
      K11=IXZ/IYY
      K12=-4.0*I*RPM/IYY
      K13=IXZ/IXX
      K14=IXZ*((IYY-IZZ)/IXX-1.0)
      K15=IXX-IYY+(IXZ**2/IXX)
      K16=I*RPM*4.0
      K17=IXX-(IXZ**2/IZZ)
      K18=IZZ-(IXZ**2/IXX)
      K20=K5*K14+K6
      K21=K5*K15+K7
      K22=K5*K16+K8
      K23=K4*(1.0-K5*K13)
      K24=K6*K13+K14
      K25=K7*K13+K15
      K26=K8*K13+K16
      K27=K11/K9
      K28=-K10/K9
      K29=-K12/K9
      TC=T/(K1*.835)
      CLL0=.25+.12*TC+.05*F
      IF (M1.LT.0.225) GO TO 16
      CLLA=(6.3+.435*TC+2.0*F)*(1.0+.333(M1-.225))
      GO TO 17
   16 CLLA=6.3+.435*TC+2.0*F
   17 CLL=CLL0+CLLA*A
      CD=CD0+K*(CLL-CLMINDG)**2+.005
      CT=T/K1
      CD=CD*(1.+.18*F*CT)
      CDA=2.*K*CLLA*(CLL-CLMINDG)
      TH=ATAN(TAN(A)*COS(PHI))
      P=-PSIDOT*SIN(TH)
      Q=PSIDOT*COS(TH)*SIN(PHI)
      R=PSIDOT*COS(TH)*COS(PHI)
      CY=(R-P*A-(G/U)*COS(TH)*SIN(PHI))/K3
      CL=(K20*Q*R+K21*P*Q+K22*Q)/K23
      CM=K27*(R**2-P**2)+K28*R*P+K29*R
      CN=(K24*Q*R+K25*P*Q+K26*Q)/K23
C ITERATION ON CN,CL,CM USING CRAMERS RULE
      DO 44 JJ=1,10
      PDOT=(K4*(CL+K5*CN)+K20*Q*R+K21*P*Q+K22*Q)/K17
      QDOT=K9*CM+K10*R*P+K11*(R**2-P**2)+K12*R
```

```
      PDOT=(K4*(CN+K13*CL)+K14*Q*R+K15*P*Q+K15*R)/K18
      DPDDCM=0.0
      DPDDCL=K4/K17
      DPDDCN=K4*K5/K18
      DQDDCM=K9
      DQDDCL=0.0
      CQDDCN=0.0
      DRDDCM=0.0
      DRDDCL=K4*K13/K19
      DRDDCN=K4/K19
      DET1=DQDDCM*(DPDDCL*DRDDCN-DRDDCL*DPDDCN)
      DELCM=QDOT*(DPDDCN*DRDDCL-DPDDCL*DRDDCN)/DET1
      DELCN=DQDDCM*(PDOT*DRDDCL-RDOT*DPDDCL)/DET1
      DELCL=DQDDCM*(RDOT*DPDDCN-PDOT*DRDDCN)/DET1
      CM=CM+DELCM
      CL=CL+DELCL
      CN=CN+DELCN
   44 CONTINUE
C MISCELLANEOUS PARAMETERS
      AWB=ALPHA+1.50
      CALL ETA (CLL,TC,F,ETATAIL)
      DEPDA=.303+.4*F+(.92-.2*F)*TC-(1.0-.2*F)*TC**2
      CLATAIL=3.208*ETATAIL
      CLAW=(.114+.052*F)
      CLW=.26+1.82*F+CLAW*AWB
      CDLW=.0052+.0044*F
      IF (TC.GT.0.1) GO TO 23
      DCMDCLL=-.244+.36*TC+(.022-.72*TC)*F
      GO TO 24
   23 DCMDCLL=-.208+.08*(TC-.1)-(.05+.16*(TC-.1))*F
   24 CONTINUE
      CLAW=CLAW*57.29578
      BETA1=SQRT(1.0-M1**2)
      XX1=(AR*M1**2)/(2.0*(AR*BETA1+2.0)*BETA1)
      LT=47.37-CG*CBAR
      LF=LT-1.30
C COMPUTE DERIVATIVES REFERENCED TO STABILITY AXES
      CTU=-3.0*CT
      CXU=CTU*COS(A)
      CZU=-(CLL*M1**2)/(1.-M1**2)-CTU*SIN(A)
      CMU=DCMDCLL*(CLL*M1**2/(1.0-M1**2))
      CXA=CLL-CDA
      CZA=-CLLA-CD
      CMA=DCMDCLL*CLLA
      CXQ=0.0
      CZQ=-2.2*ETATAIL*(LT/CBAR)
      CMQ=-2.2*ETATAIL*(LT/CBAR)**2
      CXAD=0.0
      CZAD=-2.0*ETATAIL*DEPDA*(LT/CBAR)
      CMAD=-2.0*ETATAIL*DEPDA*(LT/CBAR)**2
      CMDE=-.0287*57.29578
      CZDE=+(CBAR/LT)*ETATAIL*CMDE
      CXDE=-.080
      CYB=-(.0155-.026*F+(.030-.0508*F)*TC)*57.29578
```

```
      CLB=-(.00095-.0002*F-(.0025+.0015*F)*ALPHA)*57.29578
      CNB=(.00137+.0005*F-.0015*TC)*57.29578
      IF (TC.GE.0.2) GO TO 5
      CYBT=-.545+.65*F+(1.474-2.815*F)*TC-(1.75-2.03*F)*TC**2
      CLBT=-.0638-.0034*F-(.023+.09*F)*TC+(.09+95*F)*TC**2
      GO TO 6
    5 CYBT=-.400-.688*F
      CLBT=-.0648-.0115*F+(.009-.026*F)*(TC-.20)
    6 CNBT=(.0039-.0006*F+(.013*F-.0075)*TC)*57.29578
      CYDR=.0046*57.29578
      CLDR=(.00046-(.00021/8.0)*ALPHA)*57.29578
      CNDR=-.0016*57.29578
      CYDA=0.0
      IF (CLL.GT.1.0) GO TO 11
      CLDA=-.0602-.017*F+.029*(CLL-.80)
      GO TO 13
   11 IF (CLL.GT.1.20) GO TO 12
      CLDA=-.0544-.0172*F+(.0140+.030*F)*(CLL-1.00)
      GO TO 13
   12 CLDA=-.0516-.0114*F+.029*(CLL-1.20)
   13 CONTINUE
      IF (ALPHA.GT.2.0) GO TO 1
      CNDA=(.00058+.00002*ALPHA)*57.29578
      GO TO 2
    1 CNDA=(.00097+.000125*(ALPHA-2.0))*57.29578
    2 CYP=-.0167*CLAW-2.*A*CYBT*LF/B
      CLP=-.5300
    8 CNP=-.09*CLW-.0015-2.*A*CNBT*LF/B
      CYR=2.0*CNBT
      CLR=.28*CLW*(1.+XX1)+.0518*F-CLB-CYR*Z*COS(A)**2/B
     *+2.0*CNBT*CLBT/CYBT
      CNR=-.02*CLW**2-.3*CDOW+2.0*CNBT**2/CYBT
CORRECTION FOR MACH EFFECTS
      IF (M1.LE..225) GO TO 470
      CYB=CYB*(1.+.15*(M1-.225))
      CNB=CNB*(1.+.2*(M1-.225))
      CYBT=CYBT*(1.+.2*(M1-.225))
      CNBT=CNBT*(1.+.4*(M1-.225))
      CLP=CLP-.13*(M1-.225)
  470 CONTINUE
TRANSFORM DERIVATIVES TO BODY AXES
      CX=CT+A*CLL-CD
      CZ=-(CLL+A*CD)
      CXAB=CLL-CDA+A*(CD+CLLA)
      CZAB=A*(CLL-CDA)-CD-CLLA
      CMAB=CMA+A*CMU
      CXUB=CXU+CZA*A**2-A*(CXA+CZU)
      CZUB=CZU-CXA*A**2+A*(CXU-CZA)
      CMUB=CMU-A*CMA
      CXQB=CXQ-A*CZQ
      CZQB=CZQ+A*CXQ
      CMQB=CMQ
      CXADB=CXAD-A*CZAD
      CZADB=CZAD+A*CXAD
```

```
      CMADB=CMAD
      CXDEB=CXDE-A*CZDE
      CZDEB=CZDE+A*CXDE
      CMDEB=CMDE
      CYBB=CYB
      CLBB=CLB-A*CNB
      CNBB=CNB+A*CLB
      CYPB=CYP-A*CYR
      CLPB=CLP-A*(CLR+CNP)+CNR*A**2
      CNPB=CNP-A*(CNR-CLP)-CLR*A**2
      CYRB=CYR+A*CYP
      CLRB=CLR-A*(CNR-CLP)-CNP*A**2
      CNRB=CNR+A*(CLR+CNP)+CLP*A**2
      CYDAB=CYDA
      CYDRB=CYDR
      CLDAB=CLDA-A*CNDA
      CLDRB=CLDR-A*CNDR
      CNDAB=CNDA+A*CLDA
      CNDRB=CNDR+A*CLDR
COMPUTE COEFFICIENT MATRIX
      U0=U
      A0=A
      Q0=Q
      TH0=TH
      P0=P
      R0=R
      PHI0=PHI
      AA1=RHO*S*U0/(2.*M)
      AA2=AA1*U0
      AA3=AA1/U0
      AA4=RHO*S*CBAR/(4.*M)
      AA5=RHO*S*U0*CBAR/(2.*IYY)
      AA6=AA5*U0
      AA7=AA5*CBAR/2.
      AA8=RHO*S*B/(4.*M)
      AA9=RHO*S*U0*B
      AA10=AA9*U0/2.
      AA11=AA9*B/4.
      AA12=1.-AA4*CZADB
      AA13=IXX/IXZ-IXZ/IZZ
      AA14=IZZ/IXZ-IXZ/IXX
      AA15=RHO*S*CBAR*U0*CXADB/(4.*M)
      AA16=AA15/AA12
      AA17=1.+AA16*A0/U0
      AA18=RHO*S*U0*CBAR/(4.*M)
      AA19=1.-AA4*(CZADB-A0*CXADB)
      AA(1,6)=AA(1,7)=AA(1,8)=AA(1,9)=AA(1,10)=AA(1,12)=0.0
      AA(2,6)=AA(2,7)=AA(2,9)=AA(2,10)=AA(2,12)=0.0
      AA(3,9)=AA(3,10)=AA(3,12)=0.0
      AA(4,1)=AA(4,2)=AA(4,4)=AA(4,5)=AA(4,6)=0.0
      AA(4,9)=AA(4,10)=AA(4,11)=AA(4,12)=0.0
      AA(5,9)=AA(5,10)=AA(5,11)=0.0
      AA(6,2)=AA(6,4)=AA(6,8)=AA(6,9)=AA(6,11)=0.0
      AA(7,2)=AA(7,4)=AA(7,8)=AA(7,9)=AA(7,11)=0.0
```

```
      AA(8,1)=AA(8,2)=AA(8,5)=AA(8,6)=0.0
      AA(8,9)=AA(8,10)=AA(8,11)=AA(8,12)=0.0
      AA(9,1)=AA(9,2)=AA(9,4)=AA(9,5)=AA(9,6)= .0
      AA(9,9)=AA(9,10)=AA(9,11)=AA(9,12)=0.0
      AA(1,1)=(AA1*(2.*CX+CXUB)-A0*Q0
     *+AA16*(AA3*(2.*CZ+CZUB)+Q0/U0))/AA17
      AA(1,2)=(AA2*CXAB-U0*Q0+AA16*AA1*CZAB)/AA17
      AA(1,3)=(-A0*U0+AA15*(CXQB+CXADB*(1.+AA4*CZQB)/AA12))
     */AA17
      AA(1,4)=(-G*COS(TH0)-AA16*(G*SIN(TH0)*COS(PHI0)/U0))
     */AA17
      AA(1,5)=(U0*R0-AA16*P0)/AA17
      AA(1,8)=AA16*G*COS(TH0)*SIN(PHI0)/(U0*AA17)
      AA(2,1)=(AA3*(2.*CZ+CZUB-A0*(2.*CX+CXUB))
     *+Q0*(1.+A0**2)/U0)/AA19
      AA(2,2)=(AA1*(CZAB-A0*CXAB)+A0*Q0)/AA19
      AA(2,3)=(1.+A0**2+AA4*(CZQB-A0*CXQB))/AA19
      AA(2,4)=((G/U0)*(A0*COS(TH0)-SIN(TH0)*COS(PHI0)))/AA19
      AA(2,5)=(-P0-A0*R0)/AA19
      AA(2,8)=-((G/U0)*COS(TH0)*SIN(PHI0))/AA19
      AA(3,1)=AA5*(2.*CM+CMUB)+AA(2,1)*AA7*CMADB
      AA(3,2)=AA6*CMAB+AA(2,2)*AA7*CMADB
      AA(3,3)=AA7*(CMQB+AA(2,3)*CMADB)
      AA(3,4)=AA(2,4)*AA7*CMADB
      AA(3,5)=AA(2,5)*AA7*CMADB
      AA(3,6)=((IZZ-IXX)*R0-2.*IXZ*P0)/IYY
      AA(3,7)=((IZZ-IXX)*P0+2.*IXZ*R0-RPM*I*4.)/IYY
      AA(3,8)=AA(2,8)*AA7*CMADB
      AA(4,3)=COS(PHI0)
      AA(4,7)=-SIN(PHI0)
      AA(4,8)=-PSIDOT*COS(TH0)
      AA(5,1)=RHO*S*CY/M-R0/U0+A0*P0/U0
      AA(5,2)=P0
      AA(5,4)=-G*SIN(TH0)*SIN(PHI0)/U0
      AA(5,5)=AA1*CYBB
      AA(5,6)=AA8*CYPB+A0
      AA(5,7)=AA8*CYRB-1.
      AA(5,8)=+G*COS(TH0)*COS(PHI0)/U0
      AA(6,1)=AA9*(CL/IXZ+CN/IZZ)/AA13
      AA(6,3)=(((IXX-IYY)/IZZ+1.)*P0
     *+((IYY-IZZ)/IXZ-IXZ/IZZ)*R0+RPM*I*4./IZZ)/AA13
      AA(6,5)=AA10*(CLBB/IXZ+CNBB/IZZ)/AA13
      AA(6,6)=(AA11*(CLPB/IXZ+CNPB/IZZ)
     *+Q0*(1.+(IXX-IYY)/IZZ))/AA13
      AA(6,7)=(AA11*(CLRB/IXZ+CNRB/IZZ)
     *+Q0*((IYY-IZZ)/IXZ-IXZ/IZZ))/AA13
      AA(7,1)=AA9*(CN/IXZ+CL/IXX)/AA14
      AA(7,3)=(((IXX-IYY)/IXZ+IXZ/IXX)*P0
     *+((IYY-IZZ)/IXX-1.)*R0+RPM*I*4./IXZ)/AA14
      AA(7,5)=AA10*(CNBB/IXZ+CLBB/IXX)/AA14
      AA(7,6)=(AA11*(CNPB/IXZ+CLPB/IXX)
     *+Q0*(IXZ/IXX+(IXX-IYY)/IXZ))/AA14
      AA(7,7)=(AA11*(CNRB/IXZ+CLRB/IXX)
     *+Q0*((IYY-IZZ)/IXX-1.))/AA14
```

```
      AA(8,3)=TAN(TH )*SIN(PHI )
      AA(8,4)=PSIDOT/COS(THO)-PO*TAN(THO)
      AA(8,6)=1.0
      AA(8,7)=TAN(TH )*COS(PHIO)
      AA(9,3)=SIN(PHIO)/COS(THO)
      AA(9,4)=PSIDOT*TAN(THO)
      AA(9,7)=COS(PHIO)/COS(THO)
      AA(1,11)=(AA2*CXDEB+AA16*AA1*CZDEB)/AA17
      AA(2,11)=(AA1*(CZDEB-AO*CXDEB))/AA19
      AA(3,11)=AA6*CMDEB+AA7*CMADB*AA(2,11)
      AA(5,10)=0.0
      AA(5,12)=AA1*CYDRR
      AA(6,10)=((CLDAB/IXZ+CNDAB/IZZ)*AA10)/AA13
      AA(6,12)=((CLDRB/IXZ+CNDRB/IZZ)*AA10)/AA13
      AA(7,10)=((CNDAB/IXZ+CLDAB/IXX)*AA10)/AA14
      AA(7,12)=((CNDRB/IXZ+CLDRB/IXX)*AA10)/AA14
      CALL MEGVAL (9,AA,WN,ZETA,PD,THALF,AREAL)
WHERE WN IS NATURAL FREQUENCY,ZETA IS DAMPING
RATIO,PD IS PERIOD,THALF IS TIME TO DAMP TO HALF
AMPLITUDE AND AREAL  IS MATRIX OF TIME
CONSTANTS FOR ROLLING AND SPIRAL MODES
INTEGRATION BY RUNGE KUTTA
THIS ARRANGEMENT RECORDS EVERY 10TH VALUE
SET INITIAL VALUES OF PERTURBATIONS TO ZERO
      DO 454 KJ=1,12
  454 X(1,KJ)=0.0
      T1(1)=0.0
      T=0.
VALUE OF DELT USED WOULD BE SPECIFIED HERE
      II=1
      DO 314 J=1,12
  314 Y(J)=X(1,J)
  318 CONTINUE
      DO 316 J=1,10
  316 CALL RUNGE1 (T,MFCOUNT,DELT,Y)
      II=II+1
      DO 317 J=1,12
  317 X(II,J)=Y(J)
      T1(II)=T
      IF (II.LE.250) GO TO 318
      JJJ=II
CONVERT RADIAN MEASURE TO DEGREES
      DO 99 JJ=1,JJJ
      III=JJ
      X1(III)=X(JJ,1)
      X2(III)=X(JJ,2)*57.29578
      X3(III)=X(JJ,3)*57.29578
      X4(III)=X(JJ,4)*57.29578
      X5(III)=X(JJ,5)*57.29578
      X6(III)=X(JJ,6)*57.29578
      X7(III)=X(JJ,7)*57.29578
      X8(III)=X(JJ,8)*57.29578
      X9(III)=X(JJ,9)*57.29578
      X10(III)=X(JJ,10)*57.29578
```

```
      X11(III)=X(JJ,11)*57.29578
      X12(III)=X(JJ,12)*57.29578
   99 CONTINUE
SUBROUTINES
TAIL EFFICIENCY
      SUBROUTINE ETA (CLL,TC,F,ETATAIL)
      IF (TC.GT.0.10) GO TO 2
      IF (CLL.GT.1.20) GO TO 1
      ETATAIL=1.0+(1.25-.5*F+(1.25-.5*F)*(CLL-0.80))*TC
      GO TO 4
    1 ETATAIL=1.0+(1.75-.7*F+(.50+1.5*F)*(CLL-1.20))*(TC-0.10)
      GO TO 4
    2 IF (CLL.GT.1.20) GO TO 3
      ETATAIL=1.125-.05*F+(.125-.05*F)*(CLL-0.80)
     X+(.93-1.16*F+(.375+.5*F)*(CLL-0.80))*TC
      GO TO 4
    3 ETATAIL=1.175-.07*F+(.1625+.1250*F)*(CLL-1.20)
     X+(1.08-.96*F+(.3125+.8750*F)*(CLL-1.20))*(TC-0.10)
    4 CONTINUE
      RETURN
      END
RUNGE KUTTA
      SUBROUTINE RUNGE1 (T,MECOUNT,DT,X)
      DIMENSION P0(9),P1(9),P2(9),P3(9),X(12),X0(12),XT(12),
     *XP(12)
      T0=T
      T=T+DT
      HT=DT
      TT=T
      DO 5 I=1,12
      X0(I)=X(I)
    5 XT(I)=X(I)
      ASSIGN 6 TO K
      GO TO 20
    6 DO 7 I=1,9
    7 XP(I)=X(I)
      HT=0.5*DT
      XT(10)=X(10)
      XT(11)=X(11)
      XT(12)=X(12)
      ASSIGN 9 TO K
      GO TO 20
    9 DO 10 I=1,12
   10 XT(I)=X(I)
      TT=T0+HT
      ASSIGN 11 TO K
   20 CALL FDOT (TT,MECOUNT,XT,P0)
      DO 21 I=1,9
   21 X(I)=XT(I)+0.5*HT*P0(I)
      CALL FDOT (TT+0.5*HT,MECOUNT,X,P1)
      DO 22 I=1,9
   22 X(I)=XT(I)+HT*(.20710678l*P0(I)+.292893219*P1(I))
      CALL FDOT (TT+0.5*HT,MECOUNT,X,P2)
```

```
      DO 23 I=1,9
   23 X(I)=XT(I)+HT*(.70710678*(P2(I)-P1(I))+P2(I))
      CALL FDOT (TT+HT,MFCOUNT,X,P3)
      DO 24 I=1,9
   24 X(I)=XT(I)+HT*(P0(I)+.58578643*P1(I)+3.4142136*P2(I)
     *+P3(I))/6.0
      GO TO K,(8,9,11)
   11 DO 12 I=1,9
   12 X(I)=X(I)+(X(I)-XP(I))/15.0
      RETURN
      END
C DERIVATIVES FOR RUNGE KUTTA
      SUBROUTINE FDOT (T,MFCOUNT,X,XDOT)
      DIMENSION XDOT(9),X(12)
      COMMON AA(9,12)
C CONTROL DEFLECTION WOULD BE SPECIFIED HERE
C X(10) IS AILERON,X(11) IS ELEVATOR,X(12) IS RUDDER
C EXAMPLE: FOR ONE SECOND PULSE...
      IF (T.LT.5.0.OR.T.GE.6.0) GO TO 26
      X(11)=0.1
      GO TO 27
   20 IF (T.LT.5.0.OR.T.GE.6.0) GO TO 26
      X(10)=0.1
      GO TO 27
   21 IF (T.LT.5.0.OR.T.GE.6.0) GO TO 26
      X(12)=0.1
      GO TO 27
   26 X(10)=X(11)=X(12)=0.0
   27 CONTINUE
      DO 1 J=1,9
      XDOT(J)=0.0
      DO 3 J=1,9
      DO 2 K=1,12
    2 XDOT(J)=XDOT(J)+AA(J,K)*X(K)
    3 CONTINUE
      RETURN
      END
C SUBROUTINES TO DETERMINE EIGENVALUES
      SUBROUTINE MEGVAL (NP,A,WN,ZETA,PD,THALF,AREAL)
      DIMENSION A(9,9),RSR(9),RSI(9),ZETA(9),WN(9),PD(9)
      DIMENSION AREAL(2)
      DIMENSION THALF(9)
      DOUBLE PRECISION DA(9,9)
      IF (NP.GT.1) GO TO 180
      RSR(1)=A(1,1)
      RSI(1)=0.0
      GO TO 200
  180 DO 190 I=1,NP
      DO 190 J=1,NP
  190 DA(I,J)=A(I,J)
      CALL CHARD (NP,9,DA,0.0,RSR,RSI)
  200 DO 300 I=1,NP
      PD(I)=0.0
      WN(I)=0.0
```

```
      THALF(I)=J.C
 31^  ZETA(I)=..C
      I=1
 31   IF (I.GT.NF) GO TO 33C
      IF (RSI(I).NE.0) GO TO 320
      I=I+1
      GO TO 310
 320  WN(I)=SQRT(RSR(I)**2+RSI(I)**2)
      ZETA(I)=-RSR(I)/WN(I)
      PD(I)=6.2831853C718/(WN(I)*SQRT(1.-ZETA(I)**2))
      THALF(I)=.693/(WN(I)*ZETA(I))
      I=I+2
      GO TO 310
 330  CONTINUE
      K=1
      DO 340 I=1,NP
      IF (RSI(I).EQ.0.0.AND.RSR(I).NE.0.0) 341,340
 341  AREAL(K)=1./ABS(RSR(I))
      K=K+1
 340  CONTINUE
      IF (AREAL(2).LT.AREAL(1)) 342,343
 342  S1=AREAL(1)
      AREAL(1)=AREAL(2)
      AREAL(2)=S1
 343  CONTINUE
      DO 30 I=1,8
      K=I+1
      DO 31 J=K,9
      IF (WN(I).GE.WN(J)) GO TO 31
      R1=WN(I)
      R2=ZETA(I)
      R3=PD(I)
      R4=THALF(I)
      WN(I)=WN(J)
      ZETA(I)=ZETA(J)
      PD(I)=PD(J)
      THALF(I)=THALF(J)
      WN(J)=R1
      ZETA(J)=R2
      PD(J)=R3
      THALF(J)=R4
 31   CONTINUE
 30   CONTINUE
      RETURN
      END
      SUBROUTINE CHARC (N,NVAR,A,CRIT,RR,RI)
THIS SUBROUTINE COMPUTES THE EIGENVALUES OF A REAL MATRIX,
SYMMETRIC OR NONSYMMETRIC. THE INPUT MATRIX IS TRANSFORMED BY
SIMILARITY TRANSFORMATIONS INTO ONE OF THE FROBENIOUS FORMS
WHERE ROW 1 CONTAINS ALL BUT THE LEADING COEFF OF THE
CHARACTERISTIC ECN. THE LEADING COEFF IS 1. ACCURACY IS IN-
CREASED BY MAXIMIZING DIVISOR BY INTERCHANGING ROWS AND
COLUMNS.THE ROOTS ARE SOLVED USING A D-ALEMBERT LEMMA
WHERE ALL VALUES IN A ROW TO THE LEFT OF THE DIAG ARE LESS
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THAN INPUT CRITERIOR. PROGRAM SUBDIVIDES FROM USING PF TO
OPERATE ON 2 OR MORE LOWER POLYNOMIALS WHERE :
A IS NVAR BY NVAR DOUBLE PREC MATRIX, N IS ORDER OF MATRIX,
RR,RI ARE STORAGE ARRAYS, CRIT IS DIVISOR CRITERIA (NORMAL 0)
CHARD USES POLYRF,LEMPRT,POLYEV AND COEFER
      DIMENSION A(NVAR,NVAR),RR(1),RI(1)
      DOUBLE PRECISION COEF(11)
      DIMENSION XX(10),YY(10)
      DOUBLE PRECISION SUM,DIV,ROW(10),COL(10)
      DOUBLE PRECISION A,X,YDP
C     MATRIX NORMALIZATION FOR A SPECIAL CLASS OF PROBLEMS
C     IF N GE 20 DIVIDE ALL MATRIX ELEMENTS BY 10.0
      IF (N.LT.20) GO TO 3100
      DO 3050 I = 1,N
      DO 3050 J = 1,N
 3050 A(I,J) = A(I,J)/10.0
 3100 CONTINUE
      JACK=0
      M=N
      NR=0
    1 L=M
    2 K=L-1
      BIG=CRIT
      JJ=0
C     FIND LARGEST ROW ELEMENT TO LEFT OF DIAGONAL
      DO 10 J=1,K
      YDP = A(L,J)
      AA = DABS(YDP)
      IF (AA.LE.BIG) GO TO 10
      BIG = AA
      JJ=J
   10 CONTINUE
C     IF ALL ELEMENTS LEFT OF DIAGONAL ARE LE CRITERIA GO TO
C     COMPUTE EIGENVALUES OF REDUCED MATRIX
      IF (JJ.EQ.0) GO TO 70
C     SHIFT ROWS AND COLS IF NECESSARY
      IF (JJ.EQ.K) GO TO 40
      DO 20 J=1,M
      X = A(JJ,J)
      A(JJ,J) = A(K,J)
   20 A(K,J)=X
      DO 30 I=1,L
      X= A(I,JJ)
      A(I,JJ)=A(I,K)
   30 A(I,K)=X
   40 CONTINUE
C     MAKE SIMILARITY TRANSFORMATION ON MATRIX
      DIV=A(L,K)
C     ROW IN EFFECT IS THE LEFT OR INVERSE SIMILARITY MATRIX
C     COL IN EFFECT IS THE RIGHT            SIMILARITY MATRIX
      DO 42 J=1,M
      ROW(J)=A(L,J)
   42 COL(J)=-ROW(J)/DIV
      COL(K)=1.0/DIV
```

```
C      (ROW+I) * A  WHERE ROW IS KTH ROW,I THE IDENTITY MATRIX
       DO 50 J=1,M
       SUM=0.0D0
       DO 45 I=1,M
   45  SUM=SUM+A(I,J)*ROW(I)
   50  A(K,J)=SUM
C      (COL+I) * A  WHERE ROW IS KTH ROW,I THE IDENTITY MATRIX
C      FIRST K ROWS LESS KTH  COL.
       DO 60 I=1,K
       DO 60 J=1,M
       IF (J.EQ.K) GO TO 60
       A(I,J)=A(I,J)+A(I,K)*COL(J)
   60  CONTINUE
C      LTH  ROW
       DO 65 J=1,M
   65  A(L,J)=0.0D0
C      KTH COL
       A(L,K)=1.0D0
       DO 68 I=1,K
   68  A(I,K)=A(I,K)*COL(K)
       L=L-1
       IF (L.EQ.1) GO TO 70
       GO TO 2
C      SET UP TO COMPUTE ROOTS OF REDUCED OR FULL MATRIX
   70  CONTINUE
       IF (L.EQ.M) GO TO 200
       COEF(1) = 1.0
       J=1
       DO 80 I=L,M
       J=J+1
       COEF(J)=-A(L,I)
   80  CONTINUE
C      J BECOMES DEGREE OF POLYNOMIAL
       J=J-1
       CALL POLYRF(COEF,J,XX,YY,IERR)
       IF (IERR.NE.0) PRINT    1085, IERR
 1085  FORMAT(1H0,10X,13HPOLRF  IERR =,I8)
C      STORE J ROOTS
       DO 90 I=1,J
       NR=NR+1
       RR(NR)=XX(I)
   90  RI(NR)=YY(I)
       IF (NR.GE.N) GO TO 500
       M=N-NR
       IF (M.EQ.1) GO TO 220
       GO TO 1
C      ONE EIGENVALUE IS A DIAGONAL ELEMENT
  200  NR=NR+1
       RR(NR)=A(L,L)
       RI(NR)=0.0
  210  IF(NR.EQ.N) GO TO 500
       IF(L.EQ.2) GO TO 220
       M=N-NR
       GO TO 1
```

```
  220 NR=NR+1
      RR(NR)=A(1,1)
      RI(NR)=0.0
      GO TO 210
C     IF N GE 20 MULT. ALL ROOTS BY 10.0
  500 IF (N.LT.20) RETURN
      DO 3150 I = 1,N
      RR(I) =10.0*RR(I)
 3150 RI(I) =10.0*RI(I)
      RETURN
      END
      SUBROUTINE LEMPRT
C     THIS ROUTINE SYSTEMATICALLY FINDS A ROOT OF A POLYNOMIAL
C     USING A SIMPLE CAGING SCHEME BASED ON D-ALEMBERTS LEMMA
      COMMON /COEFFP/PP(11),M,X,Y,AP,RR,RI,IERR
      DIMENSION NFLAG(5),U(5),V(5),P(5)
      DOUBLE PRECISION PP
      EQUIVALENCE (P,P1),(P(2),P2),(P(3),P3),(P(4),P4),
     *(P(5),P5)
      L = 1
      RR = 0.0
      RI = 0.0
      SIGN = 1.0
      IFLAG = 0
      JFLAG = 0
      KFLAG = 0
      DEL = 0.5
      DDEL = 8.0
      DO  5 I = 1,5
    5 NFLAG(I)=0
   10 IF (IFLAG.LT.5) GO TO 25
      IF (JFLAG.GT.0) GO TO 25
   20 IFLAG = 0
      IF (KFLAG.LT.3) GO TO 22
      RR = RR + SIGN/10.0
      RI = RI + SIGN/13.0
      SIGN = -2.0*SIGN
   21 CONTINUE
      IF (ABS(RI).LE.1.0) GO TO 22
      SIGN = SIGN/97.0
      RR = SIGN/3.0
      RI = -SIGN
      GO TO 21
   22 KFLAG = KFLAG + 1
      DEL = DDEL*DEL
      DDEL = DDEL + 1.3
   24 NFLAG(L) = 0
      GO TO 30
   25 IFLAG = IFLAG + 1
   30 CONTINUE
      DO 40 I = 1,5
      IF (NFLAG(I).NE.0) GO TO 38
      X  = RR
      Y  = RI
```

```
      IF (I.EQ.1) GO TO 35
      IF (I.EQ.2) X = X + DEL
      IF (I.EQ.3) X = X - DEL
      IF (I.EQ.4) Y = Y + DEL
      IF (I.EQ.5) Y = Y - DEL
   35 U(I) = X
      V(I) = Y
      CALL POLYEV
      P(I) = AP
   38 NFLAG(I) = 0
   40 CONTINUE
      IF (JFLAG.GT.27) GO TO 60
      DO 45 I = 1,5
      IF (P(I).GT. 1.0E-07) GO TO 48
   45 CONTINUE
      GO TO 60
   48 DIF1 = AMAX1(P1,P2,P3,P4,P5)
      DIF2 = AMIN1(P1,P2,P3,P4,P5)
      DIF = DIF1 - DIF2
      IF (( DIF.GE.1.0).AND.(P1.LT.1.0)) GO TO 55
      IF (P1.EQ.0.0) GO TO 60
      DIF = DIF/P1
      IF (DIF.LT.0.001)GO TO 20
   55 CONTINUE
   60 CONTINUE
      DO 70 J = 1,5
      I = J
      IF (P(J).EQ.0.0) GO TO 100
   70 CONTINUE
      DIF2 = AMIN1(P2,P3,P4,P5)
      IF (P1.GT.DIF2) GO TO 80
      IF (DEL.LT.100.0E-30) RETURN
      DEL = 0.5*DEL
      XX = RR + DEL
      YY = RI + DEL
      IF ((XX.EQ.RR).AND.(YY.EQ.RI)) RETURN
      IF ((XX.EQ.RR).AND.(RI.EQ.0.0)) RETURN
      IF ((RR.EQ.0.0).AND.(RI.EQ.YY)) RETURN
      IF (JFLAG.GT.100)   GO TO 220
      JFLAG = JFLAG + 1
      NFLAG(1) = 1
      GO TO 30
   80 AMINY = P2
      N = 2
      DO 85 I=3,5
      IF (P(I).GT.AMINY) GO TO 85
      N = I
      AMINY = P(I)
   85 CONTINUE
      L = 3
      IF (N.EQ.3)  L = 2
      IF (N.EQ.4)  L = 5
      IF (N.EQ.5)  L = 4
      NFLAG(1) = 1
```

```
      NFLAG(L) = 1
      U(L) = U(1)
      U(1) = U(N)
      V(L) = V(1)
      P(L) = P1
      V(1) = V(N)
      P1   = P(N)
      RR = U(1)
      RI = V(1)
      GO TO 10
  100 RR = U(I)
      PI = V(I)
      RETURN
  220 IERRR= 2
      RETURN
      END
      SUBROUTINE POLYEV
      COMMON /COEFER/PR(11),M,X,Y,AP,RR,RI
      DOUBLE PRECISION PR,U,V,US
      U = PR(1)
      V = 0.0D0
      DO 20 I =2,M
      US = U
      U = X*U -Y*V +PR(I)
   20 V = X*V + Y*US
      AP = DABS(U) + DABS(V)
      RETURN
      END
      SUBROUTINE POLYRF(P,N,X,Y,IERR)
      COMMON /COEFER/PR(11),M,A,E,AP,RR,RI,IERRR
      DOUBLE PRECISION P(1)
      DIMENSION X(1),Y(1)
      DOUBLE PRECISION PR,D,X2,XY
      IF ((N.LT.1).OR.(N.GT.10)) GO TO 200
      IERR = 0
      IERRR= 0
      J = 1
      M = N+1
      DO 10 I = 1,M
      PR(I) = P(I)
   10 CONTINUE
      IF (N.EQ.1) GO TO 150
   15 CONTINUE
      CALL LEMBRT
      IF (RI.EQ.0.0) GO TO 40
      IF (RR.EQ.0.0) GO TO 16
      TEST = RI/RR
      IF (ABS(TEST).LT.0.000001) GO TO 40
   16 CONTINUE
      X(J) = RR
      Y(J) = RI
      IF (IERRR.NE.0) GO TO 220
      IF(J.EQ.N) RETURN
      J = J + 1
```

```
      X(J) = RR
      Y(J) = -RI
      IF(J.EQ.N) RETURN
      J = J + 1
      M = M - 2
      X2 = 2.0*RR
      XY =  -(RR*RR + RI*RI)
      DO 20 I = 2,M
      PR(I) = PR(I) + X2*PR(I-1)
      PR(I+1) = PR(I+1) + XY*PR(I-1)
   20 CONTINUE
      IF (M.EQ.2) GO TO 100
      GO TO 15
   40 CONTINUE
      X(J) = RR
      Y(J) = 0.0
      IF (IERRR.NE.0) GO TO 220
      IF (J.EQ.N) RETURN
      J = J + 1
      M = M - 1
      DO 50 I = 2,M
   50 PR(I) = PR(I) + RR*PR(I-1)
      IF (M.EQ.2) GO TO 100
      GO TO 15
  100 D = PR(1)
      X(J) = -PR(2)/D
      Y(J) = 0.0
      RETURN
  200 PRINT    1200, N
 1200 FORMAT(1H0,10X, 3HN =,I4,21HOUTSIDE LIMITS POLYRF)
      IERR=1
      RETURN
  220 IERR = IERRR
      RETURN
      END
DATA CARD FOR CONSTANT TERMS
 32.174  1745.5  132.6  13.71  3.1416  1020.0  10.09
END
```

Vita

Robert G. Lorenz was born on 25 March, 1942, in Miami, Florida. He graduated from Miami Jackson High School in June, 1959, and in that same year, entered the United States Air Force Academy at Colorado Springs, Colorado. He graduated from the Academy in June, 1963, being awarded a degree of Bachelor of Science and a commission in the United States Air Force. He spent the next six years in rated duties, amassing 3000 flying hours in the Lockeed C130A Hercules. His last tour of duty prior to entering the Air Force Institute of Technology in November, 1970, was as a pilot in the AC130 Gunship.

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